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Exponential stability in mean square of impulsive stochastic difference equations with continuous time ☆

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ABSTRACT

So far there have been few results presented on the exponential stability in mean square for impulsive stochastic difference equations with continuous time. The main aim of this work is to close this gap. Unlike earlier studies, ours does not make use of general methods such as Lyapunov methods, Itô formula methods and so forth. However, we obtain the desired result by establishing a difference inequality with continuous time. Moreover, the result obtained can be applied to stochastic difference equations, without impulsive effects, with continuous time. Finally, we construct an example to illustrate the effectiveness of our result.

1. Introduction

Difference equations with continuous time are difference equations in which the unknown function is a function of continuous time. (The term "difference equation" is usually used for difference equations with discrete time.) In practice, time is often involved as the independent variable in difference equations with continuous time. In view of this fact, we may refer to them as *difference equations with continuous time.* Difference equations with continuous time appear as natural descriptions of observed evolution phenomena in many branches of natural sciences (see, e.g., [9,10] and references therein). Deterministic and stochastic difference equations with continuous time are very popular with researchers (see, e.g., [11–14] and references therein)

Impulsive effects exist in many evolution processes in which states are changed abruptly at certain moments of time, involved in such fields as medicine and biology, economics, mechanics, electronics (see [1] and reference therein). However, in addition to impulsive effects, stochastic effects likewise exist in real systems. It is well known that a lot of dynamic systems have variable structures subject to stochastic abrupt changes, which may result from abrupt phenomena such as stochastic failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, etc. (see [3,2,4–6] and references therein). Therefore, the investigation of impulsive stochastic differential equations attracts great attention, especially as regards stability (see [7,8] and references therein).

Motivated by the results in Yang and Xu [2] concerning mean square exponential stability of impulsive stochastic difference equations with discrete time, we will, in this present work, be interested in exponential stability in mean square of stochastic difference equations with continuous time.

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2. Impulsive stochastic difference equations with continuous time

Let $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbf{P}\}$ be a probability space with a filtration satisfying the usual conditions, i.e., the filtration is continuous on the right and $\mathcal{F}_0$ contains all $\mathbf{P}$-zero sets. Let $\|\varphi\| = \sup_{s\in\Theta} |\varphi(s)|$ hold.

In the present work, we consider a class of impulsive stochastic difference equations with continuous time as follows:

$$\begin{cases} x(t+\tau) = F(t, x(t), x(t-h_1), \dots, x(t-h_m)) + G(t, x(t), x(t-h_1), \dots, x(t-h_m))\xi(t), & t \neq \tau_k, \\ x(\tau_k) = H(\tau_k, x(\tau_k^-)), \quad k = 1, 2, \dots, & t = \tau_k, \end{cases} \tag{2.1}$$

with initial condition

$$x(\theta) = \phi(\theta), \quad \theta \in \Theta = [t_0 - \tau - h_m, t_0].$$

The fixed moments of time τ_k satisfy $0 \leq \tau_1 < \tau_2 < \cdots < \tau_k < \cdots$, $\lim_{k\to\infty} \tau_k = \infty$. $\xi(t) \in R$ is a $\{\mathcal{F}_t\}$-measurable stationary and mutually independent stochastic process satisfying

$$E\xi(t) = 0, \qquad E\xi^2(t) = 1.$$

Moreover, $\xi(t)$ is independent on F and G.

Assume that

$$F : [t_0 - \tau, \infty) \times R^{m+1} \to R, \qquad G : [t_0 - \tau, \infty) \times R^{m+1} \to R, \qquad H : [0, \infty) \times R \to R.$$

In this work, we always assume that under certain conditions the system (2.1) admits a unique solution which is continuous on the right and limitable on the left. Furthermore, assume that $F(t, 0, \dots, 0) = 0$, $G(t, 0, \dots, 0) = 0$ and $H(\tau_k, 0) = 0$, $k = 1, 2, \dots$. Hence, Eq. (2.1) has a null solution.

Now, we introduce the definition of exponential stability in mean square.

Definition 2.1. The null solution of Eq. (2.1) is called exponential stability in mean square if there exist two positive constants λ and M such that

$$E|x(t)|^2 \leq M\|\varphi\|^2 e^{-\lambda t}, \quad t \geq t_0 - \tau.$$

Furthermore, we impose the following assumptions.

(H1) For any $t \geq t_0 - \tau$, there exist two nonnegative functions $a_j(t)$ and $b_j(t)$ such that

$$|F(t, x(t), x(t-h_1), \dots, x(t-h_m))| \leq \sum_{j=0}^{m} a_j(t)|x(t-h_j)|$$

and

$$|G(t, x(t), x(t-h_1), \dots, x(t-h_m))| \leq \sum_{j=0}^{m} b_j(t)|x(t-h_j)|,$$

where $h_0 = 0$.

(H2) $\sup_{t\geq 0}\{a^2(t) + b^2(t)\} = \mu < 1$, with $a(t) = \sum_{j=0}^{m} a_j(t)$ and $b(t) = \sum_{j=0}^{m} b_j(t)$.

(H3) There exist constants $d_k \geq 1$ which satisfy

$$|H(\tau_k, x(\tau_k^-))| \leq d_k|x(\tau_k^-)|, \quad k = 1, 2, \dots.$$

(H4) There is a constant $\alpha \geq 0$ such that

$$\frac{2 \ln d_k}{\tau_k - \tau_{k-1}} \leq \alpha < \lambda, \quad k = 1, 2, \dots,$$

where $\tau_0 = 0$ and λ satisfies

$$0 < \lambda \leq \frac{1}{\tau + h_m} \ln \frac{1}{\mu}.$$