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Travelling wavefronts of Belousov–Zhabotinskii system with diffusion and delay☆

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ABSTRACT

This paper is concerned with the existence, nonexistence and minimal wave speed of the travelling wavefronts of Belousov–Zhabotinskii system with diffusion and delay.
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1. Introduction

In 1959, Belousov [2] proposed the so-called Belousov–Zhabotinskii system to model the chemical reaction, and one of its simplified models takes the form as follows

$$\begin{cases} \dfrac{\partial U(x,t)}{\partial t} = \Delta U(x,t) + U(x,t)\left[1 - U(x,t) - rV(x,t)\right], \\ \dfrac{\partial V(x,t)}{\partial t} = \Delta V(x,t) - bU(x,t)V(x,t), \end{cases} \tag{1.1}$$

where $x \in \mathbb{R}, t > 0, r \in (0,1)$, b is a positive constant, and $U, V \in \mathbb{R}$ correspond to the concentration of bromic acid and bromide ion, respectively, Δ is the Laplacian operator on $\mathbb{R}$. Model (1.1), in fact, was also derived in biochemical and biological fields, see [10,11,21,22]. Recalling the chemical and biological backgrounds of (1.1), the following asymptotical boundary conditions were proposed [5,6,17,20]

$$\begin{cases} \lim_{x\to-\infty} U(x,t) = 0, \qquad \lim_{x\to-\infty} V(x,t) = 1, \\ \lim_{x\to\infty} U(x,t) = 1, \qquad \lim_{x\to\infty} V(x,t) = 0. \end{cases} \tag{1.2}$$

On the dynamics of (1.1) and (1.2), *travelling wavefront*, which takes the form of $(U(x,t), V(x,t)) = (\rho(x+ct), \varrho(x+ct))$ for some wave speed $c > 0$ and monotone wave profile function (ρ, ϱ), attracted much attention, see Murray [12], Troy [17], Ye and Wang [20] and the references cited therein. Moreover, from the viewpoint of the chemical reaction, the travelling wavefronts of (1.1) and (1.2) have significant sense, namely, the waves move from a region of higher bromic

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acid concentration to one of lower bromic acid concentration as it reduces the level of bromic ion (we can refer to Wu and Zou [19]).

It is well known that time delay seems to be inevitable in many evolutionary processes, e.g., biological science [4], therefore the time delay was incorporated into (1.1) by Wu and Zou [19], which takes the form as follows

$$\begin{cases} \dfrac{\partial U(x,t)}{\partial t} = \Delta U(x,t) + U(x,t)\left[1 - U(x,t) - rV(x,t-\tau)\right], \\ \dfrac{\partial V(x,t)}{\partial t} = \Delta V(x,t) - bU(x,t)V(x,t), \end{cases} \tag{1.3}$$

where $\tau \geq 0$ denotes a time delay. For model (1.3), some results have been established for the existence of travelling wavefronts, see, for example, Ma [8], and Wu and Zou [19]. In particular, Ma [8] proved the existence of travelling wavefronts of (1.3) with (1.2) by the upper and lower solution and Schauder's fixed point theorem if the wave speed $c > 2\sqrt{1-r}$. But for the case of $c \leq 2\sqrt{1-r}$, the existence of travelling wavefronts of (1.3) remains open. This constitutes the purpose of this paper.

We first change the variables such that $u = U$ and $v = 1 - V$, then (1.3) reduces to

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = \Delta u(x,t) + u(x,t)\left[1 - r - u(x,t) + rv(x,t-\tau)\right], \\ \dfrac{\partial v(x,t)}{\partial t} = \Delta v(x,t) + bu(x,t)\left[1 - v(x,t)\right], \end{cases} \tag{1.4}$$

and we are interested in the following asymptotic boundary conditions (see (1.2))

$$\lim_{x\to-\infty} u(x,t) = \lim_{x\to-\infty} v(x,t) = 0, \qquad \lim_{x\to\infty} u(x,t) = \lim_{x\to\infty} v(x,t) = 1. \tag{1.5}$$

Let $(u(x,t), v(x,t)) = (\phi(x+ct), \psi(x+ct))$ be the travelling wavefront of (1.4) and denote $x + ct$ by ξ, then $(\phi(\xi), \psi(\xi))$, $\xi \in \mathbb{R}$ must satisfy

$$\begin{cases} c\phi'(\xi) = \phi''(\xi) + \phi(\xi)\left[1 - r - \phi(\xi) + r\psi(\xi - c\tau)\right], \\ c\psi'(\xi) = \psi''(\xi) + b\phi(\xi)\left[1 - \psi(\xi)\right], \end{cases} \tag{1.6}$$

and the corresponding asymptotic boundary conditions as follows (see (1.5))

$$\lim_{\xi\to-\infty} \phi(\xi) = \lim_{\xi\to-\infty} \psi(\xi) = 0, \qquad \lim_{\xi\to\infty} \phi(\xi) = \lim_{\xi\to\infty} \psi(\xi) = 1. \tag{1.7}$$

By the above notations, our main concern in this paper is to investigate the monotone nondecreasing solutions of (1.6) and (1.7). In Section 2, we prove the existence of travelling wavefronts if $c \geq 2\sqrt{1-r}$ by the method of Ma [8] and an approximation argument used in [3,16]. In Section 3, the nonexistence and minimal wave speed of (1.6) and (1.7) will be proved by the theory of asymptotic spreading [7,16] and comparison principle for partial functional differential equations [9]. This is probably the first time that the nonexistence of travelling wavefronts of (1.3) has been reported, even for the case of $\tau = 0$.

2. Existence of travelling wavefronts

In this section, we shall investigate the existence of monotone solution of (1.6) and (1.7). Throughout this paper, X will be defined by

$$X = C(\mathbb{R}, \mathbb{R}^2) = \left\{u(x) | u(x) : \mathbb{R} \to \mathbb{R}^2 \text{ is uniformly continuous and bounded}\right\},$$

which is a Banach space with the super norm. For $(\phi, \psi) \in X$, denote (H_1, H_2) as follows

$$\begin{cases} H_1(\phi,\psi)(\xi) = 2\phi(\xi) + \phi(\xi)\left[1 - r - \phi(\xi) + r\psi(\xi - c\tau)\right], \\ H_2(\phi,\psi)(\xi) = b\psi(\xi) + b\phi(\xi)\left[1 - \psi(\xi)\right]. \end{cases}$$

Then (1.6) can be rewritten as

$$\begin{cases} c\phi'(\xi) = \phi''(\xi) - 2\phi(\xi) + H_1(\phi,\psi)(\xi), \\ c\psi'(\xi) = \psi''(\xi) - b\psi(\xi) + H_2(\phi,\psi)(\xi). \end{cases}$$

For $c > 0$, define constants as follows

$$\gamma_1(c) = \frac{c - \sqrt{c^2+8}}{2}, \qquad \gamma_2(c) = \frac{c + \sqrt{c^2+8}}{2},$$
$$\gamma_3(c) = \frac{c - \sqrt{c^2+4b}}{2}, \qquad \gamma_4(c) = \frac{c + \sqrt{c^2+4b}}{2}.$$

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