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Existence of positive periodic solutions for two kinds of neutral functional differential equations☆

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Abstract

In this work, we deal with a new existence theory for positive periodic solutions for two kinds of neutral functional differential equations by employing the Krasnoselskii fixed-point theorem. Applying our results to various mathematical models we improve some previous results.
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Keywords: Neutral functional differential equation; Existence; Positive periodic solution; Fixed-point theorem

1. Introduction

In this work, we investigate the existence of positive periodic solutions of the following two kinds of neutral functional differential equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t) - cx(t - \tau(t))] = -a(t)x(t) + f(t, x(t - \tau(t))) \tag{1.1}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[x(t) - c\int_{-\infty}^{0} Q(r)x(t + r)\mathrm{d}r\right] = -a(t)x(t) + b(t)\int_{-\infty}^{0} Q(r)f(t, x(t + r))\mathrm{d}r, \tag{1.2}$$

where $a(t), b(t) \in C(R, (0, \infty))$, $\tau(t) \in C(R, R)$, $f \in C(R \times R, R)$, and $a(t), b(t), \tau(t), f(t, x)$ are ω-periodic functions, $\omega > 0$ and $|c| < 1$ are constants, $Q(r) \in C((-\infty, 0], [0, \infty))$, $\int_{-\infty}^{0} Q(r)\mathrm{d}r = 1$.

It is well known that the functional differential equations (1.1) and (1.2) include many mathematical ecological models and population models (directly or after some transformation), for example:

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(1) Hematopoiesis models [7,10,11]

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t) - cx(t-\tau(t))] = -a(t)x(t) + b(t)\mathrm{e}^{-\beta(t)x(t-\tau(t))}, \tag{1.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[x(t) - c\int_{-\infty}^{0} Q(r)x(t+r)\mathrm{d}r\right] = -a(t)x(t) + b(t)\int_{-\infty}^{0} Q(r)\mathrm{e}^{-\beta(t)x(t+r)}\mathrm{d}r, \tag{1.4}$$

(2) Nicholson's blowflies models [2,4,9]

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t) - cx(t-\tau(t))] = -a(t)x(t) + b(t)x(t-\tau(t))\mathrm{e}^{-\beta(t)x(t-\tau(t))}, \tag{1.5}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[x(t) - c\int_{-\infty}^{0} Q(r)x(t+r)\mathrm{d}r\right] = -a(t)x(t) + b(t)\int_{-\infty}^{0} Q(r)x(t+r)\mathrm{e}^{-\beta(t)x(t+r)}\mathrm{d}r, \tag{1.6}$$

(3) models for blood cell production [1,5,6]

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t) - cx(t-\tau(t))] = -a(t)x(t) + b(t)\frac{x(t-\tau(t))}{1+x^n(t-\tau(t))}, \quad n > 0, \tag{1.7}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[x(t) - c\int_{-\infty}^{0} Q(r)x(t+r)\mathrm{d}r\right] = -a(t)x(t) + b(t)\int_{-\infty}^{0} Q(r)\frac{x(t+r)}{1+x^n(t+r)}\mathrm{d}r, \quad n > 0. \tag{1.8}$$

Meanwhile, since a growing population is likely to consume more (or less) food than a matured one, depending on individual species, this leads to the neutral functional differential equations. Moreover, it is well known that periodic solutions of differential equations describe the important modality of the systems. So it is important to study the existence of periodic solutions to (1.1) and (1.2).

In the work, we obtain sufficient conditions for the existence of positive periodic solutions for Eqs. (1.1) and (1.2). Our results improve and generalize the corresponding results of [11,12], where the authors discussed the special case $c = 0$ of (1.1). Meanwhile, the authors in [8] discussed Eqs. (1.1) and (1.2) with $c \in [0, 1)$. When $c \in (-1, 0)$ in (1.1) and (1.2), our results are new, and the method of proof of the existence of positive periodic solutions for the Eqs. (1.1) and (1.2) is different from that of [8].

The proof of the main results in our work is based on Krasnoselskii's fixed-point theorem. One of the key steps is to find operators T and S satisfying the conditions in the cited fixed-point theorem. To conclude the main results, firstly, we state Krasnoselskii's fixed-point theorem.

Lemma 1.1 ([13]). *Let X be a Banach space. Assume K is a bounded closed subset of X. Let*

$$T, S : K \to X$$

satisfy the following conditions:

(i) $Tx + Sy \in K$, $\forall x, y \in K$,
(ii) *S is a contractive operator,*
(iii) *T is a completely continuous operator in K.*

Then $T + S$ has a fixed point in K.

The rest of this work is organized as follows. In the second section, we give and prove our main results. As applications, in the final section, we apply our main results to some models. Besides, new results are obtained.

2. Main results

Let $X = \{x(t) : x(t) \in C(R, R), x(t) = x(t+\omega), t \in R\}$ with the norm $\|x\| = \sup_{t\in[0,\omega]} |x(t)|$; then X is a Banach space with the norm $\|\cdot\|$. Put

$$F(t,x) = \frac{f(t,x)}{a(t)} - cx, \qquad H(t,x) = \frac{b(t)}{a(t)} f(t,x) - cx.$$

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