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Reduced order observer design for nonlinear systems

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Abstract

This work is a geometric study of reduced order observer design for nonlinear systems. Our reduced order observer design is applicable for Lyapunov stable nonlinear systems with a linear output equation and is a generalization of Luenberger's reduced order observer design for linear systems. We establish the error convergence for the reduced order estimator for nonlinear systems using the center manifold theory for flows. We illustrate our reduced order observer construction for nonlinear systems with a physical example, namely a nonlinear pendulum without friction.

Keywords: Reduced order observers; Exponential observers; Nonlinear observers; Nonlinear systems

1. Introduction

The nonlinear observer design problem was introduced by Thau [1]. Over the past three decades, many significant works have been carried out on the construction of observers for nonlinear systems in the control systems literature [2–17]. This work is an extension of our recent work [14–17] on the full order observer design for nonlinear control systems.

The reduced order observer design for nonlinear systems presented in this work is a generalization of the construction of reduced order observers for linear systems devised by Luenberger [18].

To explain the concept of reduced order observers, consider the nonlinear system modelled by the equations

$$\begin{aligned} \dot{x} &= f(x) \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the *state* and $y \in \mathbb{R}^p$ is the *output* of the nonlinear system (1). For all practical situations, $p \le n$. Suppose that C has full rank, i.e. $\text{rank}(C) = p$. Then we can make a linear change of coordinates

$$\xi = \begin{bmatrix} \xi_m \\ \xi_u \end{bmatrix} = \Lambda x = \begin{bmatrix} C \\ Q \end{bmatrix} x \tag{2}$$

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where Q is chosen so that Λ is an invertible matrix. Note that $\xi_m \in \mathbb{R}^p$ and $\xi_u \in \mathbb{R}^{n-p}$. Such a choice of Q is made possible by the assumption that C has full rank.

Under the coordinates transformation (2), the plant (1) takes the form

$$\begin{bmatrix} \dot{\xi}_m \\ \dot{\xi}_u \end{bmatrix} = \begin{bmatrix} F_1(\xi_m, \xi_u) \\ F_2(\xi_m, \xi_u) \end{bmatrix}$$
$$y = \xi_m \tag{3}$$

where

$$F(\xi) = f(\Lambda^{-1}\xi).$$

The motivation for the reduced order state estimator or observer stems from the fact that in the plant model (3), the state ξ_m is directly available for measurement and hence it suffices to build an observer that estimates only the unmeasured state ξ_u. The order of such an observer will correspond to the dimension of the unmeasured state, namely $n - p \le n$. This type of observer is called a *reduced order observer* [18] and it has many important applications in design problems.

In this work, we present a reduced order exponential observer designed for a Lyapunov stable plant of the form (3). We establish that the associated estimation error decays to zero exponentially using the center manifold theory for flows [19].

This work is organized as follows. In Section 2, we give the problem statement for reduced order observer design. In Section 3, we present our main results, namely reduced order exponential order design for Lyapunov stable nonlinear systems. In Section 4, we illustrate our main results with a physical example, namely a nonlinear pendulum without friction.

2. Problem statement

In this work, we consider nonlinear plants of the form

$$\begin{bmatrix} \dot{x}_m \\ \dot{x}_u \end{bmatrix} = \begin{bmatrix} F_1(x_m, x_u) \\ F_2(x_m, x_u) \end{bmatrix}$$
$$y = x_m \tag{4}$$

where $x_m \in \mathbb{R}^p$ is the *measured state*, $x_u \in \mathbb{R}^{n-p}$ the *unmeasured state* and $y \in \mathbb{R}^p$ the *output* of the plant (4). We assume that the state vector

$$x = \begin{bmatrix} x_m \\ x_u \end{bmatrix}$$

is defined in a neighborhood X of the origin of $\mathbb{R}^n$ and $F : X \to \mathbb{R}^n$ is a $\mathcal{C}^1$ vector field vanishing at the origin.

We can define the *reduced order exponential observers* for the plant (4) as follows.

Definition 1. Consider a $\mathcal{C}^1$ dynamical system defined by

$$\dot{z}_u = G(z_u, y) \tag{5}$$

where $z_u \in \mathbb{R}^{n-p}$ and $G : \mathbb{R}^{n-p} \times \mathbb{R}^p \to \mathbb{R}^{n-p}$ is a locally $\mathcal{C}^1$ mapping with $G(0, 0) = 0$. Then the system (5) is called a **reduced order exponential observer** for the plant (4) if the following conditions are satisfied:

(O1) If $z_u(0) = x_u(0)$, then $z_u(t) = x_u(t)$ for all $t \ge 0$. (Basically, this requirement states that if the initial estimation error is zero, then the estimation error stays zero for all future time.)

(O2) For any given ϵ-ball, $B_\epsilon(0)$, of the origin of $\mathbb{R}^{n-p}$, there exists a δ-ball, $B_\delta(0)$, of the origin of $\mathbb{R}^{n-p}$ such that

$$z_u(0) - x_u(0) \in B_\delta(0) \Longrightarrow z_u(t) - x_u(t) \in B_\epsilon(0) \qquad \text{for all } t \ge 0$$

and, moreover,

$$\|z_u(t) - x_u(t)\| \le M \exp(-\alpha t)\|z_u(0) - x_u(0)\| \qquad \text{for all } t \ge 0$$