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A new existence result for impulsive dynamic equations on timescales☆

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Abstract

The paper is concerned with the boundary value problem for second-order impulsive dynamic equations on timescales. A new existence result is acquired by using a fixed point theorem due to Krasnoselskii and Zabreiko. An example is also included to illustrate our main result.

Keywords: Timescales; Jump operators; Impulsive dynamic equations; Fixed point theorem; Boundary value problem

1. Introduction

The theory of timescales and measure chains, which has recently received a lot of attention, was introduced by Stefan Hilger [9] in order to unify continuous and discrete analysis. Following Hilger, Agarwal and Bohner [1] developed the calculus on measure chains. The timescales calculus has tremendous potential for applications in mathematical models.

On the other hand, the impulsive differential equations, which arise in physics, population dynamics, economics and so on (see [4] and references therein), have become more and more important in mathematical models of real process. And the boundary value problems (hereafter to be abbreviated as BVPs) for impulsive differential equations and impulsive difference equations (see [3,7]) have received special attention from many authors in recent years.

With the development of impulsive differential equations and the theory of timescales, some authors focused their interest on the boundary value problems for impulsive dynamic equations on timescales. In 2002, Henderson [8] discussed the boundary value problem for second-order impulsive dynamic equations on timescale $\mathbb{T}$

$$\begin{cases} y^{\Delta\Delta}(t) + f(y(\sigma(t))) = 0, & t \in [0,1]_{\mathbb{T}} \setminus \{\tau\}, \\ \mathrm{Imp}(y(\tau)) = I(y(\tau)), \\ y(0) = y^{\Delta}(\sigma(1)) = 0, \end{cases} \tag{1.1}$$

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where $\text{Imp}(y(\tau)) = y(\tau^+) - y(\tau^-)$, $f \in C(\mathbb{R}, \mathbb{R}^+)$, σ is the forward jump operator, $I \in C(\mathbb{R}^+, \mathbb{R}^+)$. The author established the existence of at least two positive solutions of (1.1) via a double fixed-point theorem.

In 2004, Benchohra et al. [6] considered

$$\begin{cases} -y^{\Delta\Delta}(t) = f(t, y(t)), \quad t \in J := [0,1] \cap \mathbb{T}, t \neq t_k, k = 1, 2, \dots, m, \\ y(t_k^+) - y(t_k^-) = I_k(y(t_k^-)), \quad k = 1, 2, \dots, m, \\ y^{\Delta}(t_k^+) - y^{\Delta}(t_k^-) = \bar{I}_k(y(t_k^-)), \quad k = 1, 2, \dots, m, \\ y(0) = y(1) = 0, \end{cases} \tag{1.2}$$

where $\mathbb{T}$ is a timescale, $f : [0,1] \times \mathbb{R} \to \mathbb{R}$ is a given function, $I_k, \bar{I}_k \in C(\mathbb{R}, \mathbb{R})$, $t_k \in [0,1] \cap \mathbb{T}$ and $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = 1$. They used Schaefer's fixed point theorem and a nonlinear alternative of Leray-Schauder type to investigate the existence of solutions of (1.2). For some other works about impulsive dynamic equations, the readers are referred to [4,5].

More recently, Sun and Zhao [13] applied a fixed point theorem due to Krasnoselskii and Zabreiko [10] to the following system

$$\begin{cases} u''(t) + a(t)u'(t) + b(t)u(t) + h(t)f(u) = 0, \quad t \in (0, 1), \\ u(0) = 0, \quad u(1) = au(\eta). \end{cases} \tag{1.3}$$

They derived a new existence result for the BVP (1.3) provided that $\lim_{s\to\infty} \frac{f(s)}{s} = m$.

Inspired by the above results, in this paper, we deal with the existence of solutions for the following nonlinear second-order impulsive dynamic equations

$$\begin{cases} y^{\Delta\Delta}(t) + F(t, y(\sigma(t))) = 0, \quad t \in J := [0,b] \cap \mathbb{T}, t \neq t_k, k = 1, 2, \dots, m, \\ y(t_k^+) - y(t_k^-) = I_k(y(t_k^-)), \quad k = 1, 2, \dots, m, \\ y^{\Delta}(t_k^+) - y^{\Delta}(t_k^-) = -\bar{I}_k(y(t_k^-)), \quad k = 1, 2, \dots, m, \\ y(0) = y^{\Delta}(\sigma(b)) = 0, \end{cases} \tag{1.4}$$

where $\mathbb{T}$ is a timescale, $F : J \times \mathbb{R} \to \mathbb{R}$ is a given function, $I_k, \bar{I}_k \in C(\mathbb{R}, \mathbb{R})$, $t_k \in (0, b) \cap \mathbb{T}$, $0 < t_1 < \cdots < t_m < b$, and for each $k = 1, 2, \dots, m$, $y(t_k^+) = \lim_{h\to 0^+} y(t_k + h)$ and $y(t_k^-) = \lim_{h\to 0^-} y(t_k + h)$ represent the right and left limits of $y(t)$ at $t = t_k$ in the sense of timescales, that is, $t_k + h \in (0, b) \cap \mathbb{T}$ for each h in a neighborhood of 0 and in addition, if t_k is right-scattered, then $y(t_k^+) = y(t_k)$, whereas, if t_k is left-scattered, then $y(t_k^-) = y(t_k)$.

The notions of timescales, right-scattered, left-scattered, and the function σ will be defined later.

The main purpose of this paper is to establish the existence of solutions for the BVP (1.4) by employing a well-known fixed point theorem due to Krasnoselskii and Zabreiko [10]. The conditions imposed on F and $I_k, \bar{I}_k$ are easier to verify, and the method in this work is different from that in [3–8]. To our best knowledge, this fixed point theorem has not been applied to the boundary value problem for impulsive differential equations, so our result is new for the special case of impulsive differential equations, as well as in the general timescales setting.

The paper is formulated as follows. In Section 2, some basic knowledge about timescales is presented, and the fixed point theorem which is key to our proof is stated as well. In Section 3, we establish the new existence theorem of (1.4). And also an example is given to illustrate our main result.

2. Preliminaries

For convenience, we first introduce some basic knowledge and several definitions on timescales, which also can be found in [1,2,4,6,8,9,11,12].

By a timescale we mean a nonempty closed subset of $\mathbb{R}$ and we denote the timescale by $\mathbb{T}$ throughout the paper. Examples of timescales are $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, cantor set and so on.

The handicap that timescales are not necessarily connected is eliminated by utilizing the notion of jump operators which we shall next define.