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Further analysis on complete stability of cellular neural networks with delay[☆]

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Abstract

It is known that the complete stability of cellular neural networks with delays is very important in applications such as processing of a moving image. In this work, we utilize the Lyapunov functional method to analyse complete stability of cellular neural networks with delay. Our result is an improvement on that in [P.P. Civalleri, M. Gilli, L. Pandolfi, On stability of cellular neural networks with delay, IEEE Trans. Circuits Syst. I 40 (1993) 157–165].

Keywords: Cellular neural network; Complete stability; Equilibrium; Quadratic form

1. Introduction

Cellular neural networks (CNNs) introduced by Chua and Yang [3] have important applications in different areas including image processing [4], solving non-linear algebraic equations and some optimization problems [4]. In earlier applications, a CNN possesses a lot equilibria and must be completely stable for each input. In later applications, a CNN has to be designed to possess only one, globally stable equilibrium for each input. As electronic circuits of CNNs can be fabricated into chips by very large scale integration technology, the finite switching speed of amplifiers and communication times will introduce time delays in the interactions among the cells. Moreover, processing of moving images

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requires the introduction of time delays in the signals transmitted among the cells. These lead to a model of CNNs with delays (DCNNs) [8]. Many results for the global asymptotic stability of DCNNs have been presented; see, for example, [1,2,7] and the references therein. The complete stability of DCNNs has been studied in some papers. In [9], it was proved that DCNNs with positive cell-linking templates are completely stable almost everywhere. By using the Gauss–Seidel method and the nonsingular M-matrix, Takahashi in [10] obtained a sufficient criterion of the complete stability for DCNNs. In [5,6], it was proved that the DCNN is completely stable when the sum of the feedback matrix and delayed feedback matrix is symmetric, in [5] (or the feedback matrix and delayed feedback matrix are D-symmetric, in [6]), and the product of the length of delay and the spectral norm of the delayed feedback matrix is less than $2/3$.

Previous results on the complete stability of DCNNs have been restricted to the case of constant delays which are all the same across the network. However, delays are frequently variable in practical applications. In this work, we will discuss the complete stability of CNNs with variable delays, and obtain one sufficient condition for ensuring the complete stability of networks. Our result is improvement on that in [5].

2. Preliminaries

We deal with the model of CNNs with variable delay whose dynamics is described by the following system of functional differential equations:

$$\dot{x}(t) = -x(t) + Ay(t) + A^{\tau} y(t - \tau(t)) + u, \qquad t \geq 0, \tag{1}$$

where $A = [a_{ij}]_{n\times n}$ and $A^{\tau} = [a_{ij}^{\tau}]_{n\times n}$ are constant matrices and are called the feedback matrix and delayed feedback matrix, respectively, $x(\cdot) = [x_1(\cdot), \ldots, x_n(\cdot)]^{\mathrm{T}}$ is the state vector (the symbol T denotes transposition), $u = [u_1, \ldots, u_n]^{\mathrm{T}}$ is the input vector, $y(\cdot) = [y_1(\cdot), \ldots, y_n(\cdot)]^{\mathrm{T}}$ is the output vector with $y_i(t) = y_i(x_i(t))$ and

$$y_i(x_i) = \frac{1}{2}(|x_i + 1| - |x_i - 1|), \qquad i = 1, 2, \ldots, n, \tag{2}$$

and the delay $\tau(t)$ is differentiable and satisfies

$$0 \leq \tau(t) \leq \tau, \qquad 0 \leq \dot{\tau}(t) \leq k < 1, \qquad t \geq 0,$$

for some constants τ and k. The initial conditions $x_0(\theta) = \phi(\theta)$ associated with (1) are of the form

$$x_i(\theta) = \phi_i(\theta), \qquad \theta \in [-\tau, 0],$$

where $\phi_i(\theta) \in C([-\tau, 0], R), i = 1, \ldots, n$. Let $x(t) = x(t, \phi)$ denote a solution of (1) with $x_0(\theta) = \phi(\theta)$. Solutions of (1) are bounded due to (2) and exist on the interval $[0, \infty)$.

We say $x^* = [x_1^*, \ldots, x_n^*]^{\mathrm{T}}$ is an equilibrium if it satisfies

$$-x^* + (A + A^{\tau})y^* + u = 0,$$

where $y^* = [y_1(x_1^*), \ldots, y_n(x_n^*)]^{\mathrm{T}}$. In this work, we assume that all equilibria are isolated.

Definition. Network (1) is said to be completely stable if for any initial value ϕ, the forward trajectory $x(t, \phi)$ converges to an equilibrium.

In the following, we denote the norm of vector x by $\|x\| = (x_1^2 + \cdots + x_n^2)^{1/2}$ and the norm of matrix A by $\|A\| = (\max\{\lambda; \lambda \text{ is an eigenvalue of } A^{\mathrm{T}}A\})^{1/2}$, i.e. the spectral norm of matrix A.