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On the applicability of the Adomian method to initial value problems with discontinuities

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Abstract

In this paper we extend our results of L. Casasús, W. Al-Hayani [The decomposition method for ordinary differential equations with discontinuities, Appl. Math. Comput. 131 (2002) 245–251] to initial value problems with several types of discontinuities, giving relevant examples of linear and nonlinear cases.

Keywords: Decomposition method; Adomian's polynomials; Initial value problems (IVP); Heaviside function; Dirac delta function

1. Introduction

The Adomian decomposition method [2–5] is an approximate analytical procedure to determine the solution series to many functional equations. Although many different kinds of equations have been studied [1,6–8] there is left a great deal of work to do regarding problems of convergence and applicability of the method. We have already explored in [1] the possibilities of this method in the field of ordinary differential equations with Heaviside functions as driving terms.

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The main objective of this paper is to analyse first order initial value problems (IVP) with Heaviside functions and other cases of discontinuities. We also make use of the so called Modified Technique [9] to improve the accuracy of the method.

Let us consider the general functional equation

$$y - N(y) = f, \tag{1.1}$$

where N is a nonlinear operator, f is a known function, and we are seeking the solution y satisfying (1.1). We assume that for every f, Eq. (1.1) has one and only one solution.

The Adomian technique consists of approximating the solution of (1.1) as an infinite series

$$y = \sum_{n=0}^{\infty} y_n, \tag{1.2}$$

and decomposing the nonlinear operator N as

$$N(y) = \sum_{n=0}^{\infty} A_n, \tag{1.3}$$

where A_n are polynomials (called Adomian polynomials) of $y_0, \ldots, y_n$ [2–5] given by

$$A_n = \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i y_i\right)\right]_{\lambda=0}, \qquad n = 0, 1, 2, \ldots.$$

The proofs of the convergence of the series $\sum_{n=0}^{\infty} y_n$ and $\sum_{n=0}^{\infty} A_n$ are given in [4,10–14]. Substituting (1.2) and (1.3) into (1.1) yields

$$\sum_{n=0}^{\infty} y_n - \sum_{n=0}^{\infty} A_n = f.$$

Thus, we can identify

$$y_0 = f,$$
$$y_{n+1} = A_n(y_0, \ldots, y_n), \qquad n = 0, 1, 2, \ldots.$$

Thus all components of y can be calculated once the A_n are given. We then define the n-term approximant to the solution y by $\phi_n[y] = \sum_{i=0}^{n-1} y_i$ with $\lim_{n\to\infty} \phi_n[y] = y$.

2. Decomposition method applied to an IVP

Consider the general IVP:

$$y' + k^2 y - g(y) = \lambda f(t, y), \qquad y(0) = \alpha, \qquad 0 \le t \le T, \tag{2.1}$$

where k, λ and α are real constants, g is a (possibly) nonlinear function of y and f is a function with some discontinuity.

Applying the decomposition method as in [2–5], Eq. (2.1) can be written as

$$Ly = \lambda f(t, y) - k^2 y + N(y), \tag{2.2}$$