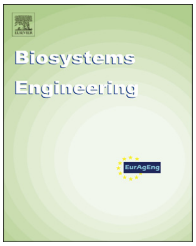
BIOSYSTEMS ENGINEERING 118 (2014) 105–114

Available online at www.sciencedirect.com

ScienceDirect

journal homepage: www.elsevier.com/locate/issn/15375110



Research Paper

Comparison of equilibrium and logarithmic models for grain drying

Daniela de C. Lopes[*], *Antonio José Steidle Neto*, *Jéssika K. Santiago*

Federal University of São João del-Rei, MG 424, km 47, Sete Lagoas, Minas Gerais, CEP 35701-970, Brazil

ARTICLE INFO

Article history:
Received 30 August 2013
Received in revised form
6 November 2013
Accepted 27 November 2013

ABSTRACT

Mathematical simulation is an important tool for optimising agricultural processes to reduce costs. Many models have been proposed and adapted to simulate grain drying. Among them, are the logarithmic or Hukill model and the equilibrium or Thorpe model, which do not require expensive solution techniques and can be used to evaluate the grain-drying behaviour regarding process time, grain temperature and variations in grain moisture content. These models were compared with experimental data found in the literature. The two models were also compared regarding predicted grain temperatures, grain moisture contents and drying times. Results showed that both models gave good prediction performance, but the Thorpe model was slightly better than the Hukill. The Thorpe model is applicable over a wider range of drying situations, including processes with variable inlet air conditions. It also provides a more fundamental understanding of the drying process. However, the logarithmic model has advantages with respect to simplicity and speed of solution.

1. Introduction

Considerable amount of grain is dried artificially either using near-ambient or high temperature air in various grain-drying systems (Aregba & Nadeau, 2007). Mathematical modelling of grain drying has been widely used to provide a better understanding of this process, to evaluate the performance of dryers, to make decisions about drying conditions or to aid in the design process of innovative systems (Bunyawanichakul, Walker, Sargison, & Doe, 2007; Jumah, 2005; Martinello, Munoz, & Giner, 2013; Martinello & Giner, 2010; Zare & Chen, 2009).

Grain-drying models can be classified as equilibrium, non-equilibrium or logarithmic types. Equilibrium models assume that equilibrium conditions exist between the grain and the drying air in each layer during a discrete period of time (Brooker, Bakker-Arkema, & Hall, 1992). Non-equilibrium models are based on the assumption that, in a deep-bed, there is no equilibrium between the drying air and grain. Thus, a set of partial differential equations is derived from the laws of heat and mass transfer and the mathematical theory of drying single solid bodies (Srivastava & John, 2002). Finally, the logarithmic models describe deep-bed drying under uniform initial and constant boundary conditions, providing an explicit analytical solution to the drying kinetics and efficiency of the drying process against time, drying air parameters and grain properties (Aregba, Sebastian, & Nadeau, 2006).

---

[*] *Corresponding author*. Tel.: +55 31 3697 2039; fax: +55 31 3697 2022.
E-mail address: danielalopes@ufsj.edu.br (D.deC. Lopes).

1537-5110/$ – see front matter © 2013 IAgrE. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.biosystemseng.2013.11.011

Nomenclature

C, D, E	constant values that depend on the stored product
c_a	specific heat of air (J kg^{-1} °C^{-1})
c_g	specific heat of grain (J kg^{-1} °C^{-1})
c_w	specific heat of water (J kg^{-1} °C^{-1})
d	index of agreement (dimensionless)
D_v	differential of latent heat of vapourisation with relation to temperature (J kg^{-1} °C^{-1})
H	half-response time of grain moisture ratio (from 1.0 to 0.5 at T_a)
h_s	differential heat of sorption (J kg^{-1})
h_v	latent heat of vapourisation of water (J kg^{-1})
i	denotes the nodes
m	denotes the temporal step
MAE	mean absolute error (h)
MBE	mean bias error (h)
M_m, M_t	grain conditions modifiers (dimensionless)
m_s	grain's dry matter loss (decimal)
M_s	rate at which dry matter is lost (s^{-1}).
N	grain depth calculated from the bulk base to the middle of the considered layer (m)
n	number of testing data (dimensionless)
O	experimental drying time (h)
P	predicted drying time (h)
q	total airflow rate (m^3 s^{-1})
Q_r	heat of oxidation of grain (J s^{-1} m^{-3})
R	humidity ratio of air (kg kg^{-1})
RMSE	root mean square error (h)
S	dryer or bin area (m^2)
T	air temperature (°C)
t	time (s)
T_a	drying air temperature (°C)
T_e	equilibrium temperature (°C)
t_p	physiological time (h)
U	grain moisture content (d.b.)
u_a	air velocity (m s^{-1})
U_e	equilibrium moisture content (decimal, d.b.)
U_s	initial moisture content (decimal, d.b.)
V_e	specific volume of air (m^3 kg^{-1})
V_m	mass airflow rate (m^3 s^{-1})
y	vertical coordinate (m)
ε	grain porosity (decimal)
Θ	grain temperature (°C)
ρ_a	density of intergranular air (kg m^{-3})
ρ_b	bulk density of the grain (kg m^{-3})

An equilibrium model, governed by partial differential equations and based on mass and energy balances, was formulated by Thorpe (1997) and presented in more detail in Thorpe (2001). This model has been used not only to simulate the aeration process, but also can be adapted for drying studies. Lopes, Martins, Melo, and Monteiro (2006) validated the Thorpe model with data from aerated maize and related some changes in the original equations in order to simplify it and decrease its computational time, without decreasing its accuracy.

Hukill (1954) developed the first logarithmic model for the analysing the drying phenomena taking place in a deep-bed. This model has been useful in approximating the drying rate of natural air and low temperature in bin dryers and was validated for different products and drying conditions, agreeing well with experimental data (Arnaud & Fohr, 1988; Li et al., 2011). The Hukill model also has been used in control strategies and didactic tools (Lopes, Martins, Steidle Neto, & Steidle Filho, 2005; Whitfield, 1988).

The two above mentioned models do not require computationally expensive solution techniques and can be used to evaluate the grain-drying behaviour regarding the process time, grain temperature and grain moisture content variations. The objective of this study was to evaluate the Thorpe and Hukill models, comparing the simulation results with experimental data found in the literature. The two models were also compared regarding predicted grain temperatures, grain moisture contents and drying times in order to evaluate differences between them. These results should contribute to a better understanding of these models and enable the most appropriate model to be selected for different drying applications.

2. Equilibrium model for grain drying (Thorpe model)

According to the simplified Thorpe model (Lopes et al., 2006), the differential equations that describe the heat and mass transfer in beds of ventilated grains can be expressed as

$$\frac{\partial\theta}{\partial t}\left\{\rho_b\left[c_g + c_w U\right] + \varepsilon\rho_a\left[c_a + R\left(c_w + \frac{\partial h_v}{\partial T}\right)\right]\right\} = \rho_b h_s\frac{\partial U}{\partial t} - u_a\rho_a\left[c_a + R\left(c_w + \frac{\partial h_v}{\partial T}\right)\right]\frac{\partial\theta}{\partial y} + \rho_b\frac{dm_s}{dt}\left(Q_r - 0.6h_v\right) \tag{1}$$

$$\frac{\partial U}{\partial t} = -\frac{\rho_a u_a}{\rho_b}\frac{\partial R}{\partial y} + 0.6\frac{dm_s}{dt}(1 + 1.66U) \tag{2}$$

where θ is the grain temperature (°C), t is the time (s), ρ_b is the bulk density of the grain (kg m^{-3}), c_g is the specific heat of grain (J kg^{-1} °C^{-1}), T is the air temperature (°C), c_w is the specific heat of water (J kg^{-1} °C^{-1}), U is the grain moisture content (d.b.), ε is the grain porosity (decimal), ρ_a is the density of intergranular air (kg m^{-3}), c_a is the specific heat of air (J kg^{-1} °C^{-1}), R is the humidity ratio of air (kg kg^{-1}), h_v is the latent heat of vapourisation of water (J kg^{-1}), h_s is the differential heat of sorption (J kg^{-1}), u_a is the air velocity (m s^{-1}), y is the vertical coordinate (m), m_s is the grain's dry matter loss (decimal) and Q_r is the heat of oxidation of grain (J s^{-1} m^{-3}).

The partial differential equations that describe the Thorpe model are coupled and solved numerically using finite difference numerical method. As the solution of one affects the solution of the other and there are non-linear terms, it is impossible to obtain closed form solutions so that the grain conditions can be expressed directly as a function of distance