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# Uncertainty analysis of a storage facility under optimal control

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Model reduction, linearisation and discretisation are standard techniques in systems theory to simplify nonlinear models. In this paper, these techniques are used on a nonlinear model of a storage facility with forced ventilation with ambient air. Consequently, weather forecasts are needed to predict the systems' trajectory. Because weather forecasts inherit uncertainty, the storage model is amenable to uncertainty. Based on a linearised version of the storage model, standard error propagation rules have been used to predict the system uncertainty analytically.

Optimal control calculates controls in such a way that a prespecified cost criterion is minimised. As the uncertainty of the storage system is subject to ventilation with outside air, uncertainty of the model state increases with increased ventilation. The model uncertainty is integrated into the cost criterion to allow a trade-off between an optimal nominal solution and a minimum variance control solution. In this respect, the predicted 95% confidence limits are kept as close as possible to the reference trajectory.



## 1. Introduction

Predictive models that are subject to disturbances generate uncertain outputs. In addition, if these disturbances are related to the inputs, like ventilation with outside air, the uncertainty of the system states increases with increasing control inputs. If optimal control strategies are used for these systems, the optimal controls are usually only valid around the nominal trajectory of the disturbances. In this paper, it is shown for a model of a storage facility of agricultural produce that it is possible to solve an optimal control problem in which the analytically calculated uncertainty is incorporated into the objective function.

The relative humidity of air acts as a driving force of evaporation from water-containing products. Water loss in stored agricultural products results in economic and quality losses (Chourasia & Goswami, 2001) and should therefore be minimised. In drying processes the opposite goal is aimed for and water needs to be removed from the product until a specific water activity is reached (e.g. Grabowski & Marcotte, 2003). In both processes, however, psychrometrics (Berliner, 1979) play an important role. Moisture content in air and relative humidity are linearly related at a constant temperature. However, with increasing temperature the maximum moisture content increases nonlinearly, i.e. the relative humidity is nonlinearly related to temperature. The actual water content is usually calculated via partial and saturated vapour pressures. There are numerous nonlinear expressions describing the saturated vapour pressure, see e.g. Sonntag (1990), Perry *et al.* (1997) and Tanner *et al.* (2002). The overall model is simplified if these relationships can be linearised over a limited temperature range,

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**Nomenclature**

| | |
|---|---|
| $a$ | parameter vector |
| $b$ | parameter vector |
| $c$ | coloured noise vector |
| $d$ | disturbance vector |
| $E$ | expectation |
| $F$ | continuous time system matrix |
| $G$ | disturbance matrix |
| $I$ | identity matrix |
| $J$ | total costs |
| $k$ | discrete-time index |
| $P$ | state covariance matrix |
| $p$ | partial pressure, Pa |
| $p_{tot}$ | atmospheric pressure, Pa |
| $p_1 \cdots p_{22}$ | parameters |
| $Q$ | covariance matrix of the disturbances |
| $T$ | temperature, °C |
| $t$ | time, s |
| $X$ | absolute humidity, kg kg$^{-1}$ |
| $x$ | state vector |
| $y$ | output vector |
| $u$ | input vector |
| $w$ | system noise |
| $\alpha$ | hatch position, fraction |
| $\gamma$ | constant |
| $\Delta$ | discretisation interval |
| $\phi$ | ventilation speed, fraction |
| $\mu$ | mean |
| $\sigma$ | standard deviation |
| $\Phi$ | discrete-time system matrix |
| $\Psi$ | terminal costs |
| $\mathcal{N}$ | normal distribution |

*Subscripts*

| | |
|---|---|
| $a$ | air around the product |
| $e$ | ambient air |
| $p$ | product |
| $ref$ | reference value |
| $s$ | saturation value |

*Superscripts*

| | |
|---|---|
| $0$ | nominal value |

which is applicable in drying processes and in storage of agricultural products.

Model linearisation of dynamical systems is usually carried out with a first-order Taylor-series expansion at specific points on the input/state trajectories. The validity of this linearisation is limited around this point. As refrigerating and cold-storage of products generally have a limited temperature range, the error made by linearisation can be calculated for the whole temperature range. The error caused by linearisation can then be compared with the errors resulting from the uncertain inputs or disturbances of the system. In cold-storage or refrigerators these disturbances are, for instance, the time-varying outside temperature.

As uncertainty plays an important role in almost every model prediction, the propagation of errors through the model becomes important. For linear or linearised systems, it holds that if the input is normally distributed then the output is also normally distributed (Schweppe, 1973). Let us illustrate this with a simple example

$$y = ax + b, \quad x \sim \mathcal{N}(\mu, \sigma), \tag{1}$$

where $a, b \in \mathbb{R}$ and $x$ is normally distributed with mean $\mu$ and standard deviation $\sigma$. Then using the rules of expected value and variance

$$E(y) = E(ax + b) = aE(x) + b, \tag{2}$$

$$\mathrm{var}(y) = \mathrm{var}(ax + b) = a^2\,\mathrm{var}(x) + b \tag{3}$$

with $E(x) = \mu$ and $\mathrm{var}(x) = \sigma^2$. Hence, $y \sim \mathcal{N}(a\mu + b, a\sigma)$, which is exact if the system is linear. In the case of a linearised system, the expressions are approximations. Once the uncertainty is known, the risks of predictive control actions can be analysed. For example, the confidence intervals can be calculated (Kabouris & Georgakakos, 1991). For nonlinear systems, no general rules for error propagation are available. Numerical approaches, such as Monte Carlo analysis, are then needed to quantify the uncertainty of the output. For bilinear systems the error propagation can only be calculated analytically if the control inputs are known in advance. The possible future control inputs can be obtained by, for instance, solving an optimal control (Stengel, 1994) problem. As such, simplifying a nonlinear model to a model linear in its states and disturbances provides a way to calculate analytically the uncertainty of the system states. This uncertainty can then be incorporated in a cost function to be used in optimal control strategies.

To illustrate the usage of uncertainty in optimal control strategies, a model of stored agricultural products is used as an example. Psychrometrics are part of this model and the main cause of nonlinearity of the system. The objective of this paper is to analyse the uncertainty of a storage model driven by weather forecasts by means of error propagation rules. In order to be able to use the generic rules of error propagation the model is simplified by linearisation, discretisation and model reduction. Given the control inputs, the predicted uncertainty is calculated and the risk analysed. Finally, instead of an additive analysis of the uncertainty, it is shown that the uncertainty can be directly incorporated in optimal control strategies.

The paper is structured as follows: in Section 2 the storage model is given and subsequently model reduction, linearisation and discretisation is performed. Furthermore, a noise model for the weather forecast is derived. General rules for error propagation are given in Section 3. Optimal control problem formulations including uncertainty of the storage model are presented in Section 4. Finally, in Section 5, some concluding remarks are given.