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Robust predictive control of polytopic uncertain systems with both state and input delays

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Abstract: A robust model predictive control algorithm for discrete linear systems with both state and input delays subjected to constrained input control is presented, where the polytopic uncertainties exist in both state matrices and input matrices. The algorithm optimizes an upper bound with respect to a state feedback control law. The feedback control law is presented based on the construction of a parameter-dependent Lyapunov function. The above optimization problem can be formulated as a LMI-based optimization. The feasibility of the optimization problem guarantees that the algorithm is robustly stable. The simulation results verify the effectiveness of the proposed algorithm.

Keywords: predictive control, robust control, time-delay system, linear matrix inequality.

1. Introduction

Model predictive control (MPC) is a widely accepted control algorithm in process industry because of its ability to cope with multivariable plants with state and control constraints. The main drawback of MPC is the difficulty to incorporate uncertainty explicitly. Kothare[1] has made progress in this direction and presented a robust MPC algorithm for two kinds of uncertain systems. This algorithm was improved by Ding[2].

In process industry, there always exist uncertain systems with time delay subjected to input control constraints. Robust stability technique of uncertain time-delay systems have been widely studied [3–5], whereas robust MPC technique was less researched. Liu[6] and Hu[7], respectively, proposed robust MPC algorithms for the uncertain systems with just state delays and with uncertain state delays. Li[8] and Chen[9] studied the guaranteed cost control for uncertain systems with both state and input delays.

The objective of this article is to extend the controlled systems from polytopic uncertain systems with just state delays to polytopic uncertain systems with both state and input delays. The other is that a single constant Lyapunov function is displaced by several Lyapunov functions each one corresponding to the different vertex of the uncertain polytope. In the algorithm, the optimization problem can be formulated as a LMI-based optimization. The simulation results can verify the effectiveness of the presented algorithm.

Notation: The notation used is fairly standard. R^n is the n-dimensional space of real-valued vectors. For a vector x and a positive definite matrix H, $\|x\|_H^2 = x^{\mathrm{T}}Hx$. The symbol$*$ induces a symmetric structure, that is to say, when H and R are symmetric matrices, then $\begin{bmatrix} H & T^{\mathrm{T}} \\ * & R \end{bmatrix} = \begin{bmatrix} H & T^{\mathrm{T}} \\ T & R \end{bmatrix}$.

2. Problem formulation

Consider a polytopic uncertain system with both state and input delays:

$$\begin{cases} x_{k+1} = A_k x_k + A_{dk} x_{k-d} + B u_k + B_{hk} u_{k-h} \\ x_k = \phi_k, k \in [-d^*, 0], d^* = \max\ \{d, h\} \end{cases} \tag{1}$$

with input constraints:

$$|u_{k+i}| \leqslant u_{\max}, i \geqslant 0 \tag{2}$$

where $x \in \Re^{n_x}$ is the state, $u \in \Re^{n_u}$ is the control input, d and h are constant integers representing the number of delay units in the state and input, respectively. $\phi(k)$ is an initial function. And we assume that $[A_k\ A_{dk}\ B_{hk}] \in \Omega$, $k \geqslant 0$. For polytopic systems, the set Ω is the polytope as follows

$$\begin{aligned}\Omega = Co\{[A_{1k}\ A_{(d1)k}\ B_{(h1)k}],\\ \cdots, [A_{Lk}\ A_{(dL)_k}\ B_{(hL)k}]\}\end{aligned} \tag{3}$$

where Co denotes the convex hull, $[A_{lk}\ A_{(dl)k}\ B_{(hl)k}]$ are vertices of the convex hull and L is the amount of models. Any $[A_k\ A_{dk}\ B_{hk}]$ within the convex set Ω is a linear combination of the vertices

$$\begin{aligned}A_k &= \sum_{l=1}^{L} \theta_{kl} A_{lk},\\ A_{dk} &= \sum_{l=1}^{L} \theta_{kl} A_{(dl)k},\\ B_{hk} &= \sum_{l=1}^{L} \theta_{kl} B_{(hl)k},\\ \text{with } &\sum_{l=1}^{L} \theta_{kl} = 1,\end{aligned}$$

$0 \leqslant \theta_{kl} \leqslant 1$. This type of systems with both state and input delays has not been paid much attention although the polytopic uncertain systems have been studied for many years.

The problem is to find the optimal control inputs which satisfy the constraints and to achieve robust performance objective as follows:

$$\min_{u_{k+i|k}, i \geqslant 0} \quad J_{k\infty} \tag{4}$$

with

$$\begin{aligned}J_{\infty}(k) = \max_{[A_{k+i}, A_d(k+i), B_h(k+i)] \in \Omega, i \geqslant 0}\\ \sum_{i=0}^{\infty} \left[\left\| x_{k+i|k} \right\|_Q^2 + \left\| u_{k+i|k} \right\|_R^2 \right]\end{aligned} \tag{5}$$

where $x_{k+i|k}$ denotes the state at time $k+i$, predicted at time k and $u_{k+i|k}$ denotes control input at time $k+i$, computed at time k. The matrices Q and R denote positive definite and semi-definite weighting matrices, respectively.

In order to reduce the infinite optimization problem (4) into a finite one, a state feedback control law is introduced:

$$u_{k+i} = F_{k+i} x_{k+i}, i \geqslant 0 \tag{6}$$

where $F_{k+i} = \sum\limits_{l=1}^{L} \theta_l(k+i) F_l$, and the following inequation is introduced for any $[A_{k+i} A_{d(k+i)} B_{h(k+i)}] \in \Omega$

$$\begin{aligned}V_{k+i+1|k} - V_{k+i|k}\\ \leqslant -[\|x_{k+i|k}\|_Q^2 + \|u_{k+i|k}\|_R^2]\end{aligned} \tag{7}$$

where V is a Lyapunov function

$$\begin{aligned}V_{k|k} = &||x_{k|k}||_{P(k)}^2 +\\ &\sum_{i=1}^{d} ||x_{k-i|k}||_S^2 +\\ &\textstyle\sum_{j=1}^{h} ||F_k x_{k-j|k}||_T^2\end{aligned} \tag{8}$$

where $P_k = \sum\limits_{l=1}^{L} \theta_{lk} P_l$,$P_k > 0$,$S > 0$ and $T > 0$. When (7) is summed from $i = 0$ to ∞, $J_{k\infty} \leqslant V_{k|k}$. Then minimization problem (4) can be transformed into minimization of $V_{k|k}$. Suppose that there exists $\gamma \in \Re$ satisfying:

$$\min J_{k\infty} \leqslant \min V_{k|k} \leqslant \min \gamma \tag{9}$$

That is to say, there must exist the following inequality:

$$\begin{aligned}V_{k|k} = &\left\| x_{k|k} \right\|_{P_k}^2 +\\ &\sum_{i=1}^{d} \left\| x_{k-i|k} \right\|_S^2 +\\ &\sum_{j=1}^{h} \left\| F_k x_{k-j|k} \right\|_T^2 \leqslant \gamma\end{aligned} \tag{10}$$

3. Main results

When input constraints are not considered, at sampling time k, the optimization problem can be written as follows:

where $j \in \{1, 2, \cdots, L\}$, $l \in \{1, 2, \cdots, L\}$.

Proof In (12), G_l is a matrix of full rank, $X_l > 0$, then

Substitute $Y_l = F_l G_l$ for (15), pre-multiply both sides of (15) with $\mathrm{diag}[X_j^{-1}\ \ G_l^{-\mathrm{T}}\ \ I\ \ I\ \ I\ \ I\ \ I\ \ I]$ and post-multiply both sides of (15) with $\mathrm{diag}[x_j^{-1}\ \ G_l^{-1}\ \ I\ \ I\ \ I\ \ I\ \ I\ \ I]$ let

$$\min_{F(k), S, T, \gamma} \gamma \text{ s.t. } (7)(9) \tag{11}$$

Define $W = \gamma S^{-1}$, $M = \gamma T^{-1}$, $X_l = \gamma P_l^{-1}$ and $F_l = Y_l G_l^{-1}$, $l \in \{1, 2, \cdots, L\}$, where Y_l and G_l are matrices with appropriate dimensions, $X_j = \gamma P_j^{-1}$, $j \in \{1, 2, \cdots, L\}$. Use Schur complements, inequality (7) can be transformed into the following LMI: