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Optimal deterministic disturbances rejection for singularly perturbed linear systems*

Zhang Baolin[1, 2], *Tang Gongyou*[1] & *Gao Dexin*[3]

1. Coll. of Information Science and Engineering, Ocean Univ. of China, Qingdao 266071, P.R. China;
2. Dept. of Information and Mathematics Sciences, Jiliang Univ, Hangzhou 310018, P.R. China;
3. Coll. of Automation and Electronic Engineering, Qingdao Univ. of Science and Technology, Qingdao 266042, P.R. China.

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Abstract: Optimal deterministic disturbances rejection control problem for singularly perturbed linear systems is considered. By using the slow-fast decomposition theory of singular perturbation, the existent and unique conditions of the feedforward and feedback composite control (FFCC) laws for both infinite-time and finite-time are proposed, and the design approaches are given. A disturbance observer is introduced to make the FFCC laws realizable physically. Simulation results indicate that the FFCC laws are robust with respect to external disturbances.

Keywords: singularly perturbed systems, deterministic disturbances, exosystem, feedforward control, optimal control, disturbance observer.

1. INTRODUCTION

It is well known that external disturbances exist in almost all practical systems, such as wind or ocean wave forces in active control systems for offshore structures[1], harmonic oscillations in flight control systems through wind shear[2] , and periodic disturbance in the disk drive[3]. In recent years, various reliable approaches with regard to the disturbance rejection and cancellation have been well documented in many literatures. Lee et al. [4] proposed the model-based iterative learning control law for time-varying constrained systems with disturbances. A robust adaptive compensator[5] was designed while an internal model structure with adaptive frequency[6] was presented to cancel the periodic disturbances. Tang et al.[7,8] developed optimal controllers via feedforward technique to reject disturbances. On the other hand, the methods applied to the analysis and feedback control of singularly perturbed systems have received considerable attention from the research community[9].

2. PROBLEM STATEMENT

Consider the standard singularly perturbed linear system described by

$$\begin{aligned} &\dot{x}(t) = A_{11}x(t) + A_{12}z(t) + B_1u(t) + Dv(t) \\ &\varepsilon\dot{z}(t) = A_{12}x(t) + A_{22}z(t) + B_2u(t), t \geq 0 \\ &x(0) = x_0, \qquad z(0) = z_0 \end{aligned} \tag{1}$$

and an exosystem is given by

$$\begin{aligned} \dot{w}(t) &= Gw(t) \\ v(t) &= Mw(t) \end{aligned} \tag{2}$$

where $x \in R^n, z \in R^m$ and $u \in R^r$ are the state vectors and control input respectively. $0 < \varepsilon \ll 1$, x_0 and z_0 are initial values of the states. $v \in R^P$ and $w \in R^q$ are the output vector and state vector of the exosystem. A_{ij}, B_i, D, G and M are constant matrices of appropriate dimensions. We assume that

(1) the pair (G, M) is observable completely.

(2) for the case of infinite-time, exosystem (2) is critically stable or asymptotically stable.

Our objective is to find optimal control law $u*$ such that the infinite-time quadratic average performance index

$$J = \lim_{T\to\infty}\frac{1}{T}\int_0^T\left\{\begin{bmatrix} x(t) \\ z(t)\end{bmatrix}^{\mathrm{T}} Q \begin{bmatrix} x(t) \\ z(t)\end{bmatrix} + u(t)^{\mathrm{T}}Ru(t)\right\}\mathrm{d}t \tag{3}$$

is to be minimized, where

$$Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^{\mathrm{T}} & Q_3 \end{bmatrix} \geq 0, \qquad R > 0$$

3. DESIGN OF FFCC LAWS

We now consider the existence and uniqueness and the physical realizable problem of the closed-loop FFCC laws. In order to present the main results, we give a useful lemma.

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Lemma 1[10] Assume that $F \in R^{m\times m}, E \in R^{n\times n}$, $L \in R^{n\times m}$, then the Sylvester equation

$$EX + XF + L = 0 \tag{4}$$

has unique solution X iff $\lambda + \mu \neq 0$ for any $\lambda \in \sigma(E)$ and $\mu \in \sigma(F)$ with $\sigma(\cdot)$ denoting the spectra of matrix.

In view of the slow-fast decomposition theory of singular perturbation, the slow subsystem is given by

$$\begin{gathered} \dot{x}_s(t) = A_0 x_s(t) + B_0 u_s(t) + Dv(t) \\ z_s(t) = -A_{22}^{-1}\left[A_{21}x_s(t) + B_2 u_s(t)\right] \\ x_s(0) = x_0 \end{gathered} \tag{5}$$

and the fast subsystem is expressed by

$$\begin{gathered} \varepsilon \dot{z}_f(t) = A_{22} z_f(t) + B_2 u_f(t) \\ z_f(0) = z_0 - z_s(0) \end{gathered} \tag{6}$$

where

$$\begin{gathered} A_0 = A_{11} - A_{12}A_{22}^{-1}A_{21} \\ B_0 = B_1 - A_{12}A_{22}^{-1}B_2 \end{gathered} \tag{7}$$

Let

$$Q[C_1 \quad C_2]^{\mathrm{T}}[C_1 \quad C_2]$$

then we can introduce the quadratic performance index for slow subsystem as

$$J_s = \lim_{T\to\infty}\frac{1}{T}\int_0^T \left[x_s^{\mathrm{T}}C_0^{\mathrm{T}}C_0x_s + 2x_s^{\mathrm{T}}C_0^{\mathrm{T}}D_0u_s + u_s^{\mathrm{T}}R_0u_s\right]\mathrm{d}t \tag{8}$$

where

$$\begin{gathered} C_0 = C_1 - C_2A_{22}^{-1}A_{21} \\ D_0 = -C_2A_{22}^{-1}B_2, R_0 = R + D_0^{\mathrm{T}}D_0 \end{gathered} \tag{9}$$

The quadratic performance index for fast subsystem (6) is given by

$$J_f = \int_0^T \left(z_f^{\mathrm{T}}Q_3z_f + u_f^{\mathrm{T}}Ru_f\right)\mathrm{d}t \tag{10}$$

The optimal control law of the subsystem to minimize performance index (10) is given by

$$u_f^* = -R^{-1}B_2^{\mathrm{T}}P_fz_f \tag{11}$$

where $P_f<0$ is the unique solution to the algebraic Riccati equation

$$A_{22}^{\mathrm{T}}P_f + P_fA_{22} - P_fS_2P_f + Q_3 = 0 \tag{12}$$

with $S_2 = B_2R^{-1}B_2^{\mathrm{T}}$. If (A_{22}, B_2, C_2) is controllable-observable completely, then Eq. (12) has unique solution $P_f>0$.

According to the maximum principle, the optimal control law of slow sub-problems (5) and (8) can be written as

$$u_s^* = -R_0^{-1}(D_0^{\mathrm{T}}C_0x_s + B_0^{\mathrm{T}}\lambda_s) \tag{13}$$

where λ_s satisfies the two-point boundary value (TPBV) problem

$$\begin{gathered} \dot{x}_s = A_sx_s - S_0\lambda_s + Hw - \\ \dot{\lambda}_s = C_0^{\mathrm{T}}(I - D_0R_0^{-1}D_0^{\mathrm{T}})C_0x_s + A_s^{\mathrm{T}}\lambda_s \\ x_s(0) = x_0, \quad \lambda_s(\infty) = 0 \end{gathered} \tag{14}$$

where

$$\begin{gathered} H = DM, S_0 = B_0R_0^{-1}B_0^{\mathrm{T}} \\ A_s = A_0 - B_0R_0^{-1}D_0^{\mathrm{T}}C_0 \end{gathered} \tag{15}$$

Let

$$\lambda_s(t) = P_sx_s(t) + \bar{P}_sw(t) \tag{16}$$

Thus, the optimal u_s^* can be rewritten as

$$u_s^* = -R_0^{-1}\left[(D_0^{\mathrm{T}}C_0 + B_0^{\mathrm{T}}P_s)x_s + B_0^{\mathrm{T}}\bar{P}_sw\right] \tag{17}$$

Substitution of Eq. (16) into Eq. (14) yields the Riccati equation

$$A_s^{\mathrm{T}}P_s + P_sA_s - P_sS_0P_s + Q_s = 0 \tag{18}$$

and the Sylvester matrix equations

$$(A_s - S_0P_s)^{\mathrm{T}}\bar{P}_s + \bar{P}_sG + P_sH = 0 \tag{19}$$

where $Q_s = C_0^{\mathrm{T}}C_0 - C_0^{\mathrm{T}}D_0R_0^{-1}D_0^{\mathrm{T}}C_0$.

Consider

$$I - D_0R_0^{-1}D_0^{\mathrm{T}} = (I + D_0R^{-1}D_0^{\mathrm{T}})^{-1} > 0$$

then we directly obtain $Q_s \geq 0$. If the triple (A_0,B_0,C_0) is controllable-observable completely, then the Riccati Eq. (18) has unique solution $P_s > 0$.

From the regulator theory of linear system, it follows that for any $\lambda \in \sigma(A_s - S_0P_s)$, the inequality $\mathrm{Re}(\lambda) < 0$ holds. On the other hand, based on the assumption (2), we have $\mathrm{Re}(\mu) \leq 0$ for any $\mu \in \sigma(G)$. This means that Sylvester Eq. (19) has unique solution $\bar{P}_s$ by Lemma 1.

Consequently, the slow optimal control law u_s^* can be obtained from Eq. (17). Further, the composite control law u_c for original problems (1) and (3) can be represented by

$$u_c(t) = k_xx(t) + k_zz(t) + k_ww(t) \tag{20}$$

where

$$\begin{gathered} k_z = -R^{-1}B_2^{\mathrm{T}}P_f \\ k_w = -(I + k_zA_{22}^{-1}B_2)R_0^{-1}B_0^{\mathrm{T}}\bar{P}_s \\ k_x = k_zA_{22}^{-1}A_{21} - (k_zA_{22}^{-1}B_2 + I)R_0^{-1}(D_0^{\mathrm{T}}C_0 + B_0^{\mathrm{T}}P_s) \end{gathered} \tag{21}$$

Based on the above analyses, we have:

Theorem 1 Consider the infinite-time optimal disturbances rejection control for singularly perturbed linear systems (1) with respect to quadratic performa-

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