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Perfectly matched layer implementation for ADI-FDTD in dispersive media

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Abstract: Alternating direction implicit finite difference time domain (ADI-FDTD) method is unconditionally stable and the maximum time step is not limited by the Courant stability condition, but rather by numerical error. Compared with the conventional FDTD method, the time step of ADI-FDTD can be enlarged arbitrarily and the CPU cost can be reduced. 2D perfectly matched layer (PML) absorbing boundary condition is proposed to truncate computation space for ADI-FDTD in dispersive media using recursive convolution(RC) method and the 2D PML formulations for dispersive media are derived. ADI-FDTD formulations for dispersive media can be obtained from the simplified PML formulations. The scattering of target in dispersive soil is simulated under sine wave and Gaussian pulse excitations and numerical results of ADI-FDTD with PML are compared with FDTD. Good agreement is observed. At the same time the CPU cost for ADI-FDTD is obviously reduced.

Key words: perfectly matched layer, alternating direction implicit, finite difference time domain, dispersive media

1. INTRODUCTION

The finite difference time domain(FDTD) method is widely used for solving electromagnetic problems[1]. The Courant stability condition must be satisfied when the method is used. Alternating direction implicit finite difference time domain (ADI-FDTD) method proposed by Namiki is based on the alternating direction implicit technique and is applied to Yee's cell to solve Maxwell's equations[2,3]. This scheme is unconditionally stable and is not dissipative. Therefore, the time step can be set arbitrarily and does not depend on the Courant stability condition, but rather on numerical error. The time step of ADI-FDTD can be set much larger than conventional FDTD for a same simulation and it means that time step number of ADI-FDTD can be less. Due to the same level complication in one step for FDTD and ADI-FDTD, the CPU cost for ADI-FDTD can be reduced. In fact the time step can not be set arbitrarily to get good accuracy because the numerical dispersion of ADI-FDTD is similar to conventional FDTD[4]. But when the minimum cell size in the computation is much smaller than the wavelength, the time step of ADI-FDTD can be much enlarged and result agree well with conventional FDTD.

Recently, ADI-FDTD has been applied to scattering problem[5], and been extended to dispersive media using Z-transform[6], auxiliary difference equations[7] and recursive convolution(RC) method[8]. In this paper, the perfectly matched layer(PML) absorbing boundary condition for ADI-FDTD in dispersive media is proposed, based on PML for dispersive media[9], ADI technique and RC method[10]. The PML formulations for Debye dispersion are derived. The PML formulations for other dispersion can be derived on the similar approach. ADI-FDTD formulations for dispersive media can be obtained easily from the PML formulations as a special case. At last the scattering of cylinder in soil is simulated under sine wave and Gaussian pulse excitations.

2. PML FOR ADI-FDTD IN DISPERSIVE MEDIA

We consider 2D TE wave and the PML equations in dispersive media with conductivity are written as follows[9]

$$\frac{\partial D_x}{\partial t} + \sigma E_x + \sigma_y E_x = \frac{\partial (H_{zx} + H_{zy})}{\partial y} \quad (1a)$$

$$\frac{\partial D_y}{\partial t} + \sigma E_y + \sigma_x E_y = -\frac{\partial (H_{zx} + H_{zy})}{\partial x} \quad (1b)$$

$$\mu \frac{\partial H_{zx}}{\partial t} + \sigma_x^* H_{zx} = -\frac{\partial E_y}{\partial x} \quad (1c)$$

$$\mu \frac{\partial H_{zy}}{\partial t} + \sigma_y^* H_{zy} = \frac{\partial E_x}{\partial y} \quad (1d)$$

For simplicity, we consider only the permittivity. In time domain, we have[10]

$$D_\eta(t) = \varepsilon_0 \varepsilon_\infty E_\eta(t) + \varepsilon_0 \int_0^t E_\eta(t-\tau)\chi_\eta(\tau)\mathrm{d}\tau \quad (2)$$

$\eta = x, y$, ε_0 is permittivity of free space, ε_∞ is the infinite frequency relative permittivity, and $\chi_\eta(\tau)$ is the electric susceptibility of PML.

Let $t = n\Delta t$ in Eq. (2), Δt is time step, D_η can be written as

$$D_\eta^n = D_\eta(n\Delta t) = \varepsilon_0\varepsilon_\infty E_\eta(n\Delta t) + \varepsilon_0\int_0^{n\Delta t} E_\eta(n\Delta t - \tau)\chi_\eta(\tau)\mathrm{d}\tau \tag{3}$$

All field components are assumed to be constant over each time interval $\Delta t/2$, Eq. (3) can be written as

$$D_\eta^n = \varepsilon_0\varepsilon_\infty E_\eta^n + \varepsilon_0\sum_{m=0}^{2n-1} E_\eta^{n-m/2}\int_{m\Delta t/2}^{(m+1)\Delta t/2}\chi_\eta(\tau)\mathrm{d}\tau \tag{4}$$

We define

$$\chi_{\eta m} = \int_{m\Delta t/2}^{(m+1)\Delta t/2}\chi_\eta(\tau)\mathrm{d}\tau \tag{5}$$

$$\Delta\chi_{\eta m} = \chi_{\eta m} - \chi_{\eta(m+1)} \tag{6}$$

From (4), we can get

$$D_\eta^{n+1/2} - D_\eta^n = \varepsilon_0(\varepsilon_\infty + \chi_{\eta 0})E_\eta^{n+1/2} - \varepsilon_0\varepsilon_\infty E_\eta^n - \varepsilon_0\sum_{m=0}^{2n-1} E_\eta^{n-m/2}\Delta\chi_{\eta m} \tag{7}$$

Substitute (7) into discrete (1) using ADI technique. The calculation of one discrete time step is performed using two procedures.

The first procedure

$$E_x^{n+1/2}(i+1/2,j) = C_{y0}E_x^n(i+1/2,j) + C_{y1}\Psi_x^n(i+1/2,j) + \frac{C_{y1}}{\Delta y}[H_{zx}^n(i+1/2,j+1/2) + H_{zy}^n(i+1/2,j+1/2) - H_{zx}^n(i+1/2,j-1/2) - H_{zy}^n(i+1/2,j-1/2)] \tag{8}$$

$$E_y^{n+1/2}(i,j+1/2) = C_{x0}E_y^n(i,j+1/2) + C_{x1}\Psi_y^n(i,j+1/2) - \frac{C_{x1}}{\Delta x}[H_{zx}^{n+1/2}(i+1/2,j+1/2) + H_{zy}^{n+1/2}(i+1/2,j+1/2) - H_{zx}^{n+1/2}(i-1/2,j+1/2) - H_{zy}^{n+1/2}(i-1/2,j+1/2)] \tag{9}$$

$$H_{zx}^{n+1/2}(i+1/2,j+1/2) = B_{x0}H_{zx}^n(i+1/2,j+1/2) - B_{x1}\cdot\frac{E_y^{n+1/2}(i+1,j+1/2) - E_y^{n+1/2}(i,j+1/2)}{\Delta x} \tag{10}$$

$$H_{zy}^{n+1/2}(i+1/2,j+1/2) = B_{y0}H_{zy}^n(i+1/2,j+1/2) + B_{y1}\cdot\frac{E_x^n(i+1/2,j+1) - E_x^n(i+1/2,j)}{\Delta y} \tag{11}$$

The second procedure

$$E_x^{n+1}(i+1/2,j) = C_{y0}E_x^{n+1/2}(i+1/2,j) + C_{y1}\Psi_x^{n+1/2}(i+1/2,j) + \frac{C_{y1}}{\Delta y}[H_{zx}^{n+1}(i+1/2,j+1/2) + H_{zy}^{n+1}(i+1/2,j+1/2) - H_{zx}^{n+1}(i+1/2,j-1/2) - H_{zy}^{n+1}(i+1/2,j-1/2)] \tag{12}$$

$$E_y^{n+1}(i,j+1/2) = C_{x0}E_y^{n+1/2}(i,j+1/2) + C_{x1}\Psi_y^{n+1/2}(i,j+1/2) - \frac{C_{x1}}{\Delta x}[H_{zx}^{n+1/2}(i+1/2,j+1/2) + H_{zy}^{n+1/2}(i+1/2,j+1/2) - H_{zx}^{n+1/2}(i-1/2,j+1/2) - H_{zy}^{n+1/2}(i-1/2,j+1/2)] \tag{13}$$

$$H_{zx}^{n+1}(i+1/2,j+1/2) = B_{x0}H_{zx}^{n+1/2}(i+1/2,j+1/2) - B_{x1}\cdot\frac{E_y^{n+1/2}(i+1,j+1/2) - E_y^{n+1/2}(i,j+1/2)}{\Delta x} \tag{14}$$

$$H_{zy}^{n+1}(i+1/2,j+1/2) = B_{y0}H_{zy}^{n+1/2}(i+1/2,j+1/2) + B_{y1}\cdot\frac{E_x^{n+1}(i+1/2,j+1) - E_x^{n+1}(i+1/2,j)}{\Delta y} \tag{15}$$

where

$$C_{\eta 0} = \frac{\dfrac{2\varepsilon_0\varepsilon_\infty}{\Delta t} - \dfrac{\sigma+\sigma_\eta}{2}}{\dfrac{2\varepsilon_0(\varepsilon_\infty+\chi_{\eta 0})}{\Delta t} + \dfrac{\sigma+\sigma_\eta}{2}} \tag{16}$$

$$C_{\eta 1} = \frac{1}{\dfrac{2\varepsilon_0(\varepsilon_\infty+\chi_{\eta 0})}{\Delta t} + \dfrac{\sigma+\sigma_\eta}{2}} \tag{17}$$

$$B_{\eta 0} = \frac{1 - \dfrac{\sigma_\eta^*\Delta t}{4\mu}}{1 + \dfrac{\sigma_\eta^*\Delta t}{4\mu}} \tag{18}$$

$$B_{\eta 1} = \frac{\dfrac{\Delta t}{2\mu}}{1 + \dfrac{\sigma_\eta^*\Delta t}{4\mu}} \tag{19}$$

$$\eta = x, y$$

For different dispersion, Ψ_η^n, $\Psi_\eta^{n+1/2}$ and $\chi_{\eta 0}$ have different expressions. For Debye dispersion, the expression for the frequency domain relative permittivity is $\varepsilon(\omega) = \varepsilon_\infty + \frac{\varepsilon_S - \varepsilon_\infty}{1 + j\omega t_0}$. The relative permittivity in PML[9] is $\varepsilon(\omega) = \varepsilon_\infty + \frac{\varepsilon_S - \varepsilon_\infty}{1 + j\omega t_0}\cdot\left(1 - \frac{\sigma_\eta t_0}{\varepsilon_0\varepsilon_S}\right)$, σ_η