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Evolution of two properties for scale-free network *

Li Shouwei & Qian Xingsan
School of Management, Univ. of Shanghai for Science and Technology, Shanghai 200093, P. R. of China
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Abstract: Fitness of node can denote its competing power and clustering denotes the transitivity of network. Because the fitness of node is uncertain or fuzzy in some social networks, an explicit form of the degree distribution on fuzzy fitness is derived within a mean field approach. It is a weighted sum of different fuzzy fitness. It can be found that the fuzzy fitness of nodes may lead to multiscaling. Moreover, the clustering coefficient of node decays as power law and clustering coefficient of network behavior not-decrease-but-increase' phenomenon after some time. Some computer simulation results of these models illustrate these analytical results.

Keywords: fuzzy fitness, clustering coefficient, power law distribution, scale-free.

1. INTRODUCTION

Because complex network structure can describe a wide variety of systems of high technological and intellectual importance, there has been a considerable interest in the growth properties of random networks[1,2]. The typical random network is a scale-free network, which connectivity distributions behave as power law. Scale-free network is an evolving network, which has many properties during evolution, for example, the fitness and clustering coefficient of node are two important properties.

Model of scale-free network was first introduced by Barabási and Albert (BA)[3,4]. In BA model networks evolve by adding new nodes and new nodes preferentially attach to highly connected nodes. So networks can self-organize into a scale-free state. As a consequence, the oldest nodes have the highest number of links, since they have the longest lifetime to accumulate them. However numerous examples indicate that in real networks a node's degree and growth rate do not depend on age alone. Real networks have a competition aspect, as each node has an intrinsic ability to compete for links at the expense of other nodes.

We already know that the clustering coefficient is higher in scale-free network than in regular network[1,2]. In the language of social networks, clustering may describe that pairs of individuals with a common acquaintance (or several) are likely to become acquainted themselves through introduction by their mutual friend(s). The clustering coefficient plays an important role in network for the connectivity and stability of network[5~7]. If a certain node in clique is deleted, the connectivity will not be changed because of the existence of transitivity, so the network has higher stability. Someone would like to ask how clustering coefficient varies during the evolution of scale-free network.

In this paper, firstly, we study the influence of fuzzy fitness on growth rates of networks based on fuzzy probability; secondly, we study the relationship of clustering coefficient and connectivity during evolution of scale-free network.

2. EVOLUTION OF FUZZY FITNESS

The competition power of node can be denoted by fitness. Therefore many models in which each node is assigned a fitness parameter η_i have emerged for studying the degree distribution of network[8~10]. In these models each node is assigned a fitness parameter which does not change in time. Each newly appearing node i is given a fitness η_i that represents its attractiveness and hence its propensity to generate new links. Fitness is chosen from some distribution $\rho(\eta)$ and links attach to nodes with

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probability proportional not just to the degree k_i of node i but to the product $\eta_i k_i$. Treating the models analytically, it is found that the power-law degree distribution is preserved for suitable parameter values, although the exponent may be affected by the distribution of fitness, and in some case there are also logarithmic corrections to the degree distribution.

The phenomenon of "rich-get-richer" appears in growing random networks described by *BA* model; if every node of growing random network is assigned to fitness, the phenomenon of "fitter-get-richer" will appear.

However, in some real networks such as social networks, the fitness of nodes is uncertain, *i. e.* the fitness is fuzzy. For example, in citation networks if a node (article) is famous we can describe its fitness "strong" or "stronger". On the World Wide Web some documents acquire a large number of edges in a very short time through a combination of good content and marking, so we can describe their fitness "high" or "higher". The fitness mentioned above is called fuzzy fitness. Based on the above idea, we describe the fuzzy fitness using fuzzy probability definition.

Definition 1 We called that the *power set* of X is the set of all subsets of X, which is denoted by $F(X)$. Assume the fitness ζ follows distribution $\rho(\zeta)$ which is defined on set X, *fuzzy fitness* $A(A \in F(X))$ of node is defined by its membership function $\mu_A(\zeta)$.

So the *probability of fuzzy fitness* is defined as follows

$$P(A) = \int \mu_A(\zeta)\mathrm{d}p = \int \mu_A(\zeta)\rho(\zeta)\mathrm{d}\zeta \qquad (1)$$

Theorem 1 From the definition of fuzzy fitness, we can obtain that probability of fuzzy fitness satisfies some properties as follows. (where $A, B \in F(X)$)

$$P(\phi) = 0, P(\Omega) = 1, 0 \leqslant P(A) \leqslant 1 \qquad (2)$$

$$\text{if } A \subseteq B, \text{ then } P(A) \leqslant P(B) \qquad (3)$$

$$P(A^c) = 1 - P(A) \qquad (4)$$

Theorem 2 The probability value of fuzzy random event is equal to the expectation of its *member ship function*.

$$P(A) = E(\mu_A(\zeta)) \triangleq E(A)$$

Arithmetic 1 **(Model of fuzzy fitness)** Evolution model of fuzzy fitness is defined in two steps.

(1) Growth: Starting with a small number m_0 of nodes, at every time step we add a new node i with fuzzy fitness A_i, $A_i \in F(X)$. Each new node i has m links that are connected to the nodes already present in the system.

(2) Preferential attachment: When choosing the nodes i to which the new node connects, we assume that the probability Π_i is proportional to the product of connectivity k_i and probability $E(A_i)$.

Theorem 3 The model of fuzzy fitness can self-organize into scale-free state as time goes on, and the degree distribution of network decays as multi-power law, $p(k) \sim \sum_A k^{-(1+\frac{C}{E(A)})}$, where C satisfies $\sum_A \frac{1}{\frac{C}{E(A)} - 1} = 1$.

Proof We also give a method to calculate analytically the probability using mean field theory, allowing us to determine exactly the exponent.

From the definition 1, A_i satisfies equation $E(A_i) = P(A_i) = \int \mu_{A_i}(\zeta)\rho(\zeta)\mathrm{d}\zeta$, ζ follows the distribution $\rho(\zeta)$ and $\mu_{A_i}(\zeta)$ is the membership function of node i.

From Preferential attachment, we can obtain that

$$\Pi_i = \frac{E(A_i)k_i}{\sum_j E(A_j)k_j} \qquad (5)$$

A node i will increase its connectivity k_i at a rate that is proportional to the probability Π_i that a new node will attach to, giving

$$\frac{\partial k_i}{\partial t} = m\frac{E(A_i)k_i}{\sum_j E(A_j)k_j} \qquad (6)$$

where, the factor m accounts for the fact that each