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Dual worth trade-off method and its application for solving multiple criteria decision making problems*

Feng Junwen
School of Economics and Management, Nanjing Univ. of Science and Technology, Nanjing 210094, P. R. China
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Abstract: To overcome the limitations of the traditional surrogate worth trade-off (SWT) method and solve the multiple criteria decision making problem more efficiently and interactively, a new method labeled dual worth trade-off (DWT) method is proposed. The DWT method dynamically uses the duality theory related to the multiple criteria decision making problem and analytic hierarchy process technique to obtain the decision maker's solution preference information and finally find the satisfactory compromise solution of the decision maker. Through the interactive process between the analyst and the decision maker, trade-off information is solicited and treated properly, the representative subset of efficient solutions and the satisfactory solution to the problem are found. The implementation procedure for the DWT method is presented. The effectiveness and applicability of the DWT method are shown by a practical case study in the field of production scheduling.

Keywords: decision theory, multiple criteria decision making, trade-off analysis, duality theory, analytic hierarchy process.

1. INTRODUCTION

The multiple criteria decision making (MCDM) problem has two remarkable properties: incommensurability and inter-contradictoriness among objectives. By incommensurability among objectives is meant that there is no uniform measuring standard for each objective, hence it is difficult to compare the each other. By inter-contradictoriness is meant that if some solution is taken to make some objective value better, then it is possible to make another or several other objectives worse. For the MCDM problems, dozens of decision making methods have been developed, but only goal programming method is applied better and more broadly[1]. Y. Y. Haimes studied and developed a multi-objective decision making method by means of the concept of the trade-off ratio-surrogate worth trade-off (SWT) method[2]. The SWT method has at least the following several limitations in applications. (1) Difficulties in generating the set or a representative subset of efficient solutions. In SWT method, inequality constraint method is suggested, but the operations are not easy. (2) Improper treatment for the trade-off information. The trade-off ratios, in fact, are relatively the two-two compared values of objectives in analytic hierarchy process (AHP) or analytic network process (ANP)[3].

Since the work by Y. Y. Haimes[3], several possible efforts related to improving SWT method have been made[4~7]. But no further satisfactory work has been done. As far as the above limitations of the SWT method are concerned, this paper presents a new decision making method labeled dual worth trade-off (DWT) method, which partly overcomes these shortcomings.

2. TRADE- OFF RATIO AND ITS DUALITY

Consider the following multiple criteria decision making problem:

(MCDM) $\max f(x)$ s. t. $g(x) \leqslant b,\ x \in X$

where $f=(f_1, f_2, \cdots, f_n)$, $g=(g_1, g_2, \cdots, g_m)$, and $b=(b_1, b_2, \cdots, b_m)$ is a constant, X is the decision space.

The ε-constraint inequality problem of MCDM with regard to the reference objective function f_i is defined by

$$\varepsilon\text{-}MP_i \qquad \max \quad f_i(x)$$

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$$\text{s. t.} \begin{cases} g_j(x) \leqslant b_j, \ j = 1, 2, \cdots, m \\ f_k(x) \geqslant \varepsilon_k, \ k = 1, 2, \cdots, n, k \neq i \\ x \in X \end{cases}$$

Definition (Trade-off) Suppose that x^* is the efficient solution of MCDM, and there is another efficient solution x^0 in some neighborhood $N(x^*)$ at x^*. If f_k and f_i are continuously differential at $x=x^*$, then there exists some direction vector d^* and some scalar $\alpha \in [0, \alpha^0]$, such that $x^0 = x^* + \alpha d^*$. We define the global trade-off ratio of f_i with respect to f_k along direction d^* at x^* to be

$$t_{ik} = \lim_{\alpha \to 0} \frac{f_i(x^* + \alpha d^*) - f_i(x^*)}{f_k(x^* + \alpha d) - f_k(x^*)} = \frac{\nabla f_i(x^*) \cdot d^*}{\nabla f_k(x^*) \cdot d^*}$$

From $\nabla f_i(x^*) \cdot d^* \approx \Delta f_i(x^*)$, $\nabla f_k(x^*) \cdot d^* \approx \Delta f_k(x^*)$, we get $t_{ik} \approx \Delta f_i(x^*)/\Delta f_k(x^*)$. Furthermore, if d^* is a direction vector, then there exists $\alpha^0 > 0$ such that $f_l(x^* + \alpha d^*) = f_l(x^*)$ for $l=1, 2, \cdots, n$, $l \neq k$ and $0 \leqslant \alpha \leqslant \alpha^0$. In this case, we call t_{ik} partial trade-off ratio and denote it by l_{ik}.

The dual programming of ε-MP_i is

$$(\varepsilon\text{-}MD_i) \qquad \min f_i(x) - \sum_{k \neq i} \lambda_{ik}(f_k(x) - \varepsilon_k) + \sum_{j=1}^{m} \mu_{ij}(g_j(x) - b_j)$$
$$\text{s. t. } x \in X, \lambda_{ik} \geqslant 0, \mu_{ij} \geqslant 0, k = 1, 2, \cdots, n; k \neq i, j = 1, 2, \cdots, m$$

The relationship between the partial trade-off ratio l_{ik} and the optimal solution x^*, λ_{ik}^* ($k=1, 2, \cdots, n$, $k \neq i$), and μ_{ij}^* ($j=1, 2, \cdots, m$) of(ε-MP_i)is described by the following duality result.

Duality Theory[8] Given the ε^0, let x^* solve $\varepsilon^0 - MD_i$. If

(1) x^* is a regular point of $\varepsilon^0 - MP_i$, that is, $\nabla f_k(x^*)$, $k \in I$, and $\nabla g_j(x^*)$, $j \in J$, are linearly independent, where $I = \{k: f_k(x^*) = \varepsilon_k^0, k \neq i, k \in \{1, 2, \cdots, n\}\}$, and $J = \{j: g_j(x^*) = b_j, j = 1, 2, \cdots, m\}$;

(2) x^*, λ_{ik}^* ($k \neq i$, $k=1, 2, \cdots, n$) and μ_{ij}^* ($j=1, 2, \cdots, m$) satisfy the second sufficiency conditions, that is,

$$\lambda_{ik}^*(f_k(x^*) - \varepsilon_k^0) = 0 \quad \text{for } k = 1, 2, \cdots, n, \ k \neq i$$
$$\mu_{ij}^*(g_j(x^*) - b_j) = 0 \quad \text{for } j = 1, 2, \cdots, m$$
$$\nabla f_i(x^*) - \sum_{k \neq i} \lambda_{ik}^* \nabla f_k(x^*) + \sum_{j=1, 2, \cdots, n} \mu_{ij}^* \nabla g_j(x^*) = 0$$

And the Hesse matrix $L_i(x^*)$ of $l_i(x)$ at x^*, where

$$l_i(x) = f_i(x) - \sum_{k \neq i} \lambda_{ik}^*(f_k(x^*) - \varepsilon_k^0) - \sum_{j=1, 2, \cdots, n} \mu_{ij}^*(b_j - g_j(x))$$
$$L_i(x) = F_i(x) - \sum_{k \neq i} \lambda_{ik}^* F_k(x) + \sum_{j=1, 2, \cdots, n} \mu_{ij}^* G_j(x))$$

And for any function $h: R^m \to R$, $H(x) = [h_{ij}(x)]_{m \times m}$, $h_{ij}(x) = \partial^2 h(x)/\partial x_i \partial x_j$, $i, j = 1, 2, \cdots, n$, is positive definite on the subspace $M' = \{y: \nabla f_k(x^*)y = 0, \nabla g_j(x^*)y = 0, k \in I, j \in J\}$;

(3) There are no degenerate constraints at x^* for ε^0-MP_i, that is,

$$\lambda_{ik}^* > 0 \text{ for } k \in I \text{ and } \mu_{ij}^* > 0 \text{ for } j \in J \ ;$$

then $\lambda_{ik}^* = -\partial f_i(x^*)/\partial \varepsilon_k^0$ for $k \neq i$, $k=1, 2, \cdots, n$, $\mu_{ij}^* = \partial f_i(x^*)/\partial b_j$, for $j=1, 2, \cdots, m$.

From this theorem, furthermore, if $f_k(x^*) = \varepsilon_k^0$ for $k \neq i$, $k=1, 2, \cdots, n$, then

$$\lambda_{ik}^* = -\partial f_i(x^*)/\partial f_k(x^0)$$
$$k \neq i, \ k = 1, 2, \cdots, n$$

That is, λ_{ik}^* is just the negative value of the partial trade-off ratio of f_i with respect to f_k at x^*.

Before we go further, the following result is easily shown.

Theorem Let $x^* \in X$, and $\varepsilon_i = f_i(x^*)$ for $i=1, 2, \cdots, n$. Then x^* is the efficient solution of MCDM if and only if for any $i=1, 2, \cdots, n$, x^* is the optimal solution of ε-MP_i.

3. DUAL WORTH TRADE-OFF DECISION MAKING METHOD

Based on the above discussion, with respect to MCDM, the following multiple criteria decision making method —DWT method is developed.