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Contour representation based on wedgelet*

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Abstract: Aiming at the shortcomings of the existing wedgelet compression arithmetics, a novel contour-representing algorithm based on wedgelets is presented in this paper. Firstly the input image is binarized and the most optimized wedgelets are found by means of quadtree framework. Then the contours are reconstructed by applying the wedgelets, the data volume is compressed, and the shortcomings of the contour representation based on normal wavelet are ameliorated, the better effect for the visualization is obtained, too.

Key words: wedgelet, binarization, quadtree.

1. INTRODUCTION

Recently, visual data has acquired a predominant role in the exchange of information in many fields of communications. However, because of the inconvenience in the storage and distribution of the large amount of data, image compression begins to play an important role. Although it has emerged as one of the preeminent tool for image compression at present, yet wavelets[1] still have obvious shortages. In general, a natural image contains edges, smooth regions and textures[2]. Despite their widespread success in image processing, such as processing smooth regions and textures, wavelet-based algorithms have significant shortcomings in their treatment of edge structure in images. For instance, representing a long, straight edge in the image using wavelet basis functions is quite difficult. Not only because it takes many significant wavelet coefficients to represent the edge, but also the coefficients have the complicated inter-relationships[3]. Ignoring these relationships during processing will influence the image to a great degree, if not, "ringing" around the edges in the final image will occur. Moreover, the larger the rate is, the more prominent the ringing is, such as Fig. 1(b) and Fig. 1(c). As one of the most important features of image geometrical structure, the representation quality of the image contours impacts the effect of the whole image representation, namely the image visual, so it is worth considering and treating to the compression and representation of the contour. However, without considering the geometrical structure, the prevenient algorithms only optimize between distortion and rate, which is the key reason resulting in the ringing.

Towards it, Donoho[4] introduced the multiscale wedgelet framework to treat the contour, and used wavelet-based code to compress smooth regions and textures. Although this algorithm acts well in the solving of ringings, the representation contours look disorderly and unsystematic because of the existing of interfering signals. Then Romberg et al[5, 6] improved the former algorithm by adding a new penalty function. Though the representation effort has some advance relative to Donoho's algorithm, it is still unsatisfactory for the existence of interfering signals. Aiming at those shortages, some improvements are presented in this paper: binarizing input image to eliminate the interfering signals, so contours are more apparent; defining a new penalty function, to predigest the calcu-

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lating complexity and keep wedgelets border upon scale coherent. Consequently, the final contours are smoother, and the representation effort is better.

(a) the original image (b) the compressed image by bior at rate 3.60 (c) the compressed image by bior at rate 11.65

Fig. 1 The compressed image using the wavelet

2. WEDGELET

Let I be an image supported on $[0,1]^2$, and $S_{j,k}\in[0,1]^2$, $k=\{k_1,k_2\}$, be a dyadic block at scale $j\in Z$, $S_{j,k}=[k_1/2^j,(k_1+1)/2^j]\times[k_2/2^j,(k_2+1)/2^j]$, $0\leqslant k_1,k_2\leqslant 2^j$.

The j determines the size of $S_{j,k}$; $k=\{k_1,k_2\}$, determines the orientation of $S_{j,k}$ in the $[0,1]^2$.

A wedgelet w is a function on a square $S_{j,k}$ that is piecewise constant on either side of a line L through $S_{j,k}$. Four parameters are needed to define w: two parameters for L—in this paper it uses the points (v_1,v_2) where L intersects the perimeter of $S_{j,k}$; and the value w take over (c_a) and below (c_b) L (see Fig. 2(a)). The (v_1,v_2) determine the orientation of w, while c_a and c_b determine its profile. For each orientation $(v_1,v_2)\in\nu$(ν is a finite set of orientations), $S_{j,k}$ is divided into two regions R_a (the region above the line L defined by (v_1,v_2)) and R_b (the region below L). The profile of the wedgelet at orientation (v_1,v_2) is calculated by averaging $I(S_{j,k})$ over these regions

$$c_a = \text{Average}(I(S_{j,k})/R_a) \qquad (1)$$

$$c_b = \text{Average}(I(S_{j,k})/R_b) \qquad (2)$$

A function that is constant over all of $S_{j,k}$ is called a degenerate wedgelet, namely $c_a=c_b$. It can think of such a function as a wedgelet where the line L does not pass through $S_{j,k}$. See Fig. 2, (a), (b) are the different wedgelet. It can infer the local geometrical structure $I(S_{j,k})$ (I in the region $S_{j,k}$) by finding the orientation for which $W(v_1,v_2,c_a,c_b)$ is a good approximation to $I(S_{j,k})$[7].

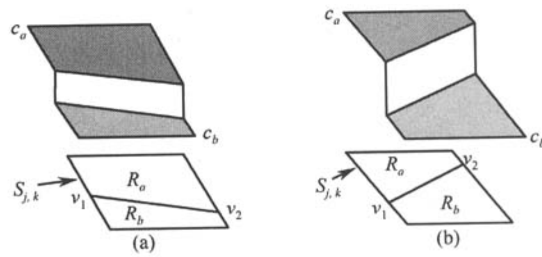

Fig. 2 The wedgelet

A wedgelet is a piecewise constant function on a dyadic square with a linear discontinuity, see figure 2. Each dyadic square contains a simple line. If all the dyadic square are incorporated in the some order, the line in the dyadic square must be approximated to the contours (see Fig. 8). Obviously, it can succinctly represent the edge of the image by the wedgelet. The contour of the image which varies slowly in the region can be approximated by the wedgelet at the big dyadic square; the contour of the image which varies quickly in the region can be approximated by the wedgelet at the small dyadic square.

3. THE CONTOUR REPRESENTATION BASED ON WEDGELET

Similar to the common wavelet representation[8], there are two components in the multiscale wedgelet framework: decomposition and representation. The multiscale wedgelet decomposition (MWD) divides the image into dyadic blocks at different scales and projects these image I blocks onto wedgelets at various orientations; while the multiscale wedgelet representation (MWR) is an approximation of the image I built out of wedgelet from the MWD, and the representation algorithm is optimized by the approximation, parsimony and geometry here. See Fig. 3, it is the procession this paper decomposes and represents the contour by the wedgelet. First of all, binarizing the input image; afterward, the binarized image is projected onto wedgelets at various dyadic blocks and then the most optimized wedgelets are found; lastly, the contours of the input image are represented by it. Figure 3 (d) is an example that the representation contour is an approximation of the contour of the original image.