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Edge agreement of second-order multi-agent system with dynamic quantization via the directed edge Laplacian

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ABSTRACT

This work explores the edge agreement problem of second-order multi-agent systems with dynamic quantization under directed communication. To begin with, by virtue of the directed edge Laplacian, we propose a model reduction representation of the closed-loop multi-agent system depending on the spanning tree subgraph. Considering the limitations of the finite bandwidth channels, the quantization effects of second-order multi-agent systems under directed graph are considered. The static quantizers generally contain a fixed quantization interval and infinite quantization level, which are, to some extent, inefficient and impractical. To further reduce the bit depth (number of bits available) and to obtain better precision, the dynamic quantized communication strategy referring to zooming in-zooming out scheme is required. Based on the reduced model associated with the *essential edge Laplacian*, the asymptotic stability of second-order multi-agent systems under dynamic quantized effects with only finite quantization level can be guaranteed. Finally, the simulation of altitude alignment of micro air vehicles is provided to verify the theoretical results.

1. Introduction

The coordination control problem of multi-agent systems has received increasing amounts of attention recently. Network topology and information flow have turned out to be an important concern of such issue, as the constraints on communication have a considerable impact on the performance of multi-agent systems. Early efforts on such problem primarily focused on the assumption that agents can obtain precise information through local communication as in [1,2]. However, in practical, only a finite amount of information data can be transmitted among neighbors at each time instant, since the digital channels are always subjected to a limited channel capacity.

To cope with the limitations of the finite bandwidth channels, information data are generally processed by quantizers. The spectral properties of the incidence matrix is employed to carry out the convergence analysis of multi-agent systems for both uniform quantizer and logarithmic quantizer in [3]. Further, in [4], by using stochastic gossiping algorithm, the explicit relationship between the convergence rate and the communication topology is revealed for uniform quantizers. Note that, the above-mentioned static quantizers require infinite quantization level which is impractical; and the fixed quantization interval always leads to practical stability rather than asymptotic stability. Therefore, the dynamic quantizer with finite quantization level is more of practical significance. In [5], the coding/decoding strategies based on zooming in-zooming

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out scheme for dynamic uniform quantizer are proposed to maintain average consensus. Based on dynamic encoding and decoding scheme, [6] provides an explicit relationship of the asymptotic convergence and the network parameters, especially, the quantization level. In addition, the authors of [6] also provide a way to reduce the number of transmitting bits along each digital channel down to merely one bit. Most recently, extensions of [6] are further discussed in view of the quantized consensus over directed networks [7–9]. Note that, these methods are mainly devised for first-order multi-agent systems. Nevertheless, these methods may lead to a dramatically different coordination behavior while considering second-order dynamics, even though the agents are coupled through similar network topology [10]. To the best of authors' knowledge, there are still little works to explore the dynamic quantization effects on second-order dynamics. [11] proposes a quantized-observer based encoding–decoding scheme for second-order multi-agent systems with limited information, which shows that exponential asymptotic synchronization can be achieved with 2-bit quantizer for connected graph. The zooming in-zooming out strategy is proposed to achieve asymptotic average consensus for double-integrator multi-agent systems with dynamically quantized information transmission in [12]. However, the above-mentioned literatures only consider the quantization effects associated with undirected graph. Since the quantization may cause undesirable oscillating behavior under directed topology [13], the scenario considering directed graph is still very challenging.

In this paper, we are going to extend our previous work of [14] to deal with the challenging scenario that second-order multi-agent systems with dynamic quantization under directed graph. Note that the analysis of the node agreement (consensus problem) has matured, but the work related to the edge agreement [15,16] has not been deeply studied yet. Since quantized measurements bring enormous challenges to the analysis of the synchronization behavior of the second-order multi-agent systems, we are going to explore more details about this term by virtue of the reduced edge agreement model. The main contributions of this paper contain three aspects. First, a model reduction representation of the closed-loop multi-agent system is derived based on the observation that the co-spanning tree subsystem can be served as an internal feedback. By utilizing the reduced edge agreement model, the analysis of the whole system can be extremely simplified. Second, contrary to [11] and [12], the challenging scenario that second-order multi-agent systems with dynamic quantization under directed graph is considered. Third, by using the zooming in-zooming out scheme, the asymptotic stability of second-order multi-agent systems under dynamic quantized effects can be guaranteed with only finite quantization level and the theoretic results are verified by achieving altitude alignment of micro air vehicles.

The rest of the paper is organized as follows: preliminaries and some related notions are proposed in Section 2. The dynamic quantized edge agreement with second-order multi-agent systems under directed graph is studied in Section 3. The simulation results are provided in Section 4, while the last section draws the conclusion.

2. Basic notions and preliminary results

In this section, some basic notions in graph theory and preliminary results about the synchronization of multi-agent systems under quantized information are briefly introduced.

2.1. Graph and matrix

In this paper, we use $|\cdot|$ and $\|\cdot\|$ to denote the Euclidean norm and 2-norm for vectors and matrices, respectively. Denote by I_n the identity matrix and by $\mathbf{0}_n$ the zero matrix in $\mathbb{R}^{n\times n}$. Let $\mathbf{0}$ be the column vector with all zero entries. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a digraph of order N specified by a node set $\mathcal{V}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ with size L. The set of neighbors of node i is denoted by $\mathcal{N}_i = \{j : e_k = (j, i) \in \mathcal{E}\}$. The adjacency matrix of $\mathcal{G}$ is defined as $A_{\mathcal{G}} = \left[a_{ij}\right] \in \mathbb{R}^{N\times N}$ with nonnegative adjacency elements $a_{ij} > 0 \Leftrightarrow (j, i) \in \varepsilon$. The degree matrix $\Delta_{\mathcal{G}} = \left[\Delta_{ij}\right]$ is a diagonal matrix with $[\Delta_{ii}] = \sum_{j=1}^{N} a_{ij}$, $i = 1, 2, \ldots, N$, and the graph Laplacian of the weighted digraph $\mathcal{G}$ is defined by $L_{\mathcal{G}}(\mathcal{G}) = \Delta_{\mathcal{G}} - A_{\mathcal{G}}$ whose eigenvalues will be ordered and denoted as $0 = \lambda_0 = \cdots = \lambda_1 < \lambda_2 \le \cdots \le \lambda_N$. Denote by $\mathcal{W}(\mathcal{G})$ the $L \times L$ diagonal matrix of w_k, for $k = 1, 2, \ldots, L$, where w_k represents the weight of $e_k = (j, i) \in \mathcal{E}$. The incidence matrix $E(\mathcal{G})$ for a digraph is a $\{0, \pm 1\}$-matrix with rows and columns indexed by nodes and edges of $\mathcal{G}$, respectively, such that for edge $e_k = (j, i) \in \mathcal{E}$, $[E(\mathcal{G})]_{jk} = +1$, $[E(\mathcal{G})]_{ik} = -1$ and $[E(\mathcal{G})]_{lk} = 0$ for $l \ne i, j$. The in-incidence matrix $E_{\odot}(\mathcal{G}) \in \mathbb{R}^{N\times L}$ is a $\{0, -1\}$ matrix with rows and columns indexed by nodes and edges of $\mathcal{G}$, respectively, such that for an edge $e_k = (j, i) \in \mathcal{E}$, $[E_{\odot}(\mathcal{G})]_{lk} = -1$ for $l = i$, $[E_{\odot}(\mathcal{G})]_{lk} = 0$ otherwise. The weighted in-incidence matrix $E_{\odot}^{w}(\mathcal{G})$ can be defined as $E_{\odot}^{w}(\mathcal{G}) = E_{\odot}(\mathcal{G})\,\mathcal{W}(\mathcal{G})$. As thus, the graph Laplacian of $\mathcal{G}$ has the following expression [14]: $L_{\mathcal{G}}(\mathcal{G}) = E_{\odot}^{w}(\mathcal{G})E(\mathcal{G})^T$. The weighted edge Laplacian of a directed graph $\mathcal{G}$ can be defined as [14]

$$L_e(\mathcal{G}) := E(\mathcal{G})^T E_{\odot}^{w}(\mathcal{G}). \tag{1}$$

A directed path in digraph $\mathcal{G}$ is a sequence of directed edges and a directed tree is a digraph in which, for the root i and any other node j, there is exactly one directed path from i to j. A spanning tree $\mathcal{G}_{\mathcal{T}} = (\mathcal{V}, \mathcal{E}_1)$ of a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a directed tree formed by graph edges that connect all the nodes of the graph; a cospanning tree $\mathcal{G}_{\mathcal{C}} = (\mathcal{V}, \mathcal{E} - \mathcal{E}_1)$ of $\mathcal{G}_{\mathcal{T}}$ is the subgraph having all the vertices of $\mathcal{G}$ and exactly those edges that are not in $\mathcal{G}_{\mathcal{T}}$. Graph $\mathcal{G}$ is called *quasi-strongly connected* if and only if it has a directed spanning tree [17].

Lemma 1 (*[14]*). *For a quasi-strongly connected graph $\mathcal{G}$, the graph Laplacian $L_{\mathcal{G}}(\mathcal{G})$ and the edge Laplacian $L_e(\mathcal{G})$ have the same $N-1$ nonzero eigenvalues, which are all in the open right-half plane.*