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Improved results for stochastic stabilization of a class of discrete-time singular Markovian jump systems with time-varying delay ☆

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ABSTRACT

In this paper, the problem of stochastic stabilization for a class of discrete-time singular Markovian jump systems with time-varying delay is investigated. By using the Lyapunov functional method and delay decomposition approach, improved delay-dependent sufficient conditions are presented, which guarantee the considered systems to be regular, causal and stochastically stabilizable. Finally, some numerical examples are provided to illustrate the effectiveness of the obtained methods.

1. Introduction

Singular systems, which are also called implicit systems, semi-state systems, descriptor systems or generalized state-space systems, are more convenient than state-space systems in the description of many dynamical systems. This is due to the fact that a singular system model contains not only differential equations (or difference equations) but also algebraic equations. The applications of this class of systems can be found extensively in chemical engineering systems, economy systems, circuit systems, power systems and other areas [1–3]. Since the regularity and impulse-free (or causality) need to be checked usually for singular systems and not to be considered for state-space ones, the study of singular systems is often much more challenging. During the past several decades, considerable attention has been dedicated to the study of singular systems, see, e.g., [4–8] (for continuous-time case) and [9–12] (for discrete-time case).

As well known, many practical systems may experience random abrupt changes in their structures and parameters. Such changes are usually caused by, e.g., component failures or repairs, sudden environmental disturbances and changes in subsystem interconnections. These systems are usually modeled as Markovian jump systems. Therefore, in the past few decades, Markovian jump systems have been investigated widely, see, e.g., [13–17] (for continuous-time case) and [18–21] (for discrete-time case).

Compared with the singular systems and Markovian jump systems, singular Markovian jump systems (SMJSs) are more general. Therefore, the SMJSs can describe more complete dynamical models. It is well known that mathematical models arising from many practical systems always involve time delay and time delay is often a source of instability and poor

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performance. Much attention has been focused on the problems of stability and stabilization for continuous-time SMJSs with time delay, see, e.g., [22–26]. The problems of stability and stabilization for the discrete-time SMJSs with time delay also have been studied in some works. [27] investigated the problems of stability and stabilization for discrete-time SMJSs with invariant time delay. The problems of stability and stabilization for the discrete-time SMJSs with time-varying delay were studied in [28]. [29] further studied the problem of robust stabilization for discrete-time uncertain SMJSs with time-varying delay. However, the results obtained in [28,29] for the stabilization of discrete-time SMJSs with time-varying delay have conservatism to some extent, which leave room for further improvement. This motivates the present work of this paper.

This paper is concerned with the stochastic stabilization problem for a class of discrete-time SMJSs with time-varying delay. Some delay-dependent sufficient conditions are presented to guarantee the considered systems to be regular, causal and stochastically stabilizable. The stochastic stabilization problem for the discrete-time SMJSs with time-varying delay have been studied in [28,29]. Compared with [28,29], contributions of this paper can be summarized as follows. First, the delay decomposition approach is employed to deal with the problem of stochastic stabilization for discrete-time SMJSs with time-varying delay. Second, in [28], although the use of the matrix $[\varepsilon_1 I_n\ \varepsilon_2 I_n]$ simplifies the analysis, it leads to some conservatism in the presented stabilization criteria. A more general matrix $[\varepsilon_1 X\ \varepsilon_2 X]$ is introduced in this paper. Third, the obtained results of this paper are less conservative than those of [28,29]. Some numerical examples are presented to illustrate the effectiveness and the advantage of the methods proposed in this paper.

Notations: R^n denotes the n-dimensional Euclidean space. $R^{m\times n}$ is the set of all $m \times n$ real matrices. For a real symmetric matrix X, $X > 0$ (respectively, $X \geq 0$) means that X is positive definite (respectively, semi-positive definite). $\mathcal{Z}$ stands for the set of non-negative integer numbers. $\mathcal{C}$ denotes the set of complex numbers. $\mathcal{E}\{\cdot\}$ stands for the mathematical expectation operator. I is the identity matrix of appropriate dimensions. The symbol "$*$" denotes the symmetric elements in a symmetric matrix. $\lambda_{\max}(\cdot)$ means the largest eigenvalue of a matrix. The superscript "T" denotes the transpose of a matrix or a vector. diag$\{\cdots\}$ stands for a block-diagonal matrix. mod (m, n) denotes the remainder of $\frac{m}{n}$.

2. Preliminaries

Consider the discrete-time SMJSs with time-varying delay as follows:

$$\begin{cases} Ex_{k+1} = (A(r_k) + \Delta A(r_k))x_k + (A_d(r_k) + \Delta A_d(r_k))x_{k-d(k)} + (B(r_k) + \Delta B(r_k))u_k, \\ x_k = \phi(k), \quad k = -d_2, \ldots, -1, 0, \end{cases} \tag{1}$$

where $x_k \in R^n$ is the state vector of the system, $\phi(k)$ is a given initial condition, and $u_k \in R^m$ is the control input. $d(k)$ denotes the time-varying delay and satisfies $0 < d_1 \leq d(k) \leq d_2$, where d_1 and d_2 are known positive integers. $E \in R^{n\times n}$ is a known real constant matrix and we assume that E is singular and satisfies $0 < \text{rank}(E) = r < n$. $\{r_k, k \in \mathcal{Z}\}$ is a discrete-time homogeneous Markov chain taking values in a finite set $\mathcal{L} = \{1, 2, \ldots, N\}$ with transition probability matrix $\Pi = (\pi_{ij})_{N\times N}$ given by

$$P\{r_{k+1} = j | r_k = i\} = \pi_{ij}, \tag{2}$$

where $0 \leq \pi_{ij} \leq 1$ is the transition probability from mode i to mode j and $\sum_{j=1}^{N} \pi_{ij} = 1$. For each $r_k \in \mathcal{L}$, the matrices $A(r_k)$, $A_d(r_k)$ and $B(r_k)$ are known real constant matrices with appropriate dimensions.

In system (1), for each $r_k \in \mathcal{L}$, $\Delta A(r_k)$, $\Delta A_d(r_k)$ and $\Delta B(r_k)$ are time-varying uncertainties with appropriate dimensions, and assumed to be of the following form:

$$[\Delta A(r_k)\ \Delta A_d(r_k)\ \Delta B(r_k)] = C(r_k)F(k, r_k)\left[U_1(r_k)\ U_2(r_k)\ U_3(r_k)\right], \tag{3}$$

where $C(r_k)$, $U_1(r_k)$, $U_2(r_k)$ and $U_3(r_k)$ are known real constant matrices with appropriate dimensions. $F(k, r_k) \in R^{q\times s}$, for each $r_k \in \mathcal{L}$, is an unknown real time-varying matrix satisfying

$$F^T(k, r_k)F(k, r_k) \leq I. \tag{4}$$

The uncertainties $\Delta A(r_k)$, $\Delta A_d(r_k)$ and $\Delta B(r_k)$ are said to be admissible if both (3) and (4) hold. If $\Delta A(r_k) \equiv 0$, $\Delta A_d(r_k) \equiv 0$ and $\Delta B(r_k) \equiv 0$, the system (1) reduces to the system as follows:

$$\begin{cases} Ex_{k+1} = A(r_k)x_k + A_d(r_k)x_{k-d(k)} + B(r_k)u_k, \\ x_k = \phi(k), \ k = -d_2, \ldots, -1, 0. \end{cases} \tag{5}$$

For brevity, in the sequel, we denote $A(r_k) = A_i$, $A_d(r_k) = A_{di}$, $B(r_k) = B_i$ and $F(k, r_k) = F(k, i)$ for each $r_k = i \in \mathcal{L}$, and the other symbols are denoted similarly.

In this paper, the memoryless mode-dependent feedback controller with the following form is considered:

$$u_k = K(r_k)x_k, \tag{6}$$

where $K(r_k)$, for each $r_k \in \mathcal{L}$, is a constant matrix to be determined.

Some definitions should be introduced firstly to facilitate the following discussion.