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Stability analysis of discrete-time switched systems with time-varying delays via a new summation inequality

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ABSTRACT

This paper proposes a new summation inequality, which improves the conservatism in the stability analysis for discrete-time systems with time-varying delay. In order to show the effectiveness of the proposed inequality, which provides general lower bound of the summation quadratic term of the form $\sum_{s=a}^{b} x^T(s)Mx(s)$, a delay-dependent stability criterion for such systems is derived within the framework of linear matrix inequalities (LMIs). Going one step forward, the proposed inequality is applied to a stability problem in discrete-time switched systems with time-varying delays. The advantages of employing the proposed summation inequality are illustrated via three numerical examples.

1. Introduction

During the last two decades, the problem of the stability analysis of various systems with time-delay has been extensively investigated in control society [1–5]. In the field of delay-dependent stability analysis, one major issue is to obtain the maximum delay bounds for guaranteeing the delay-dependent stability of time-delay systems.

Naturally, in order to obtain improved results, various methods were introduced. Simply put, the model transformation [6,7], the null sum terms [8,9], the positive definiteness [10], Park's inequality [11], Jensen's inequality [12,13], the reciprocally convex approach [14], and others [15–22]. For the integral quadratic terms of the form $\mathcal{J}_C(x) = \int_a^b \dot{x}^T(s)M\dot{x}(s)ds$ $(M > 0)$, Wirtinger-based inequality was presented by [23] to reduce the gap of its lower bound than the one obtained by Jensen inequality. Specifically, on the continuous-time systems, since the finding of the lower bounds for the integral quadratic term such as $\mathcal{J}_C(x)$ is one of the major topics, improved stability criteria are often drawn upon the bounding methods in [14,23].

On this wise, the lower bound for bounds for the summation quadratic term such as $\mathcal{J}_D(\Delta x) = \sum_{s=a}^{b} \Delta x^T(s)M\Delta x(s)$ $(M > 0)$, where $\Delta x(s) = x(s+1) - x(s)$, is expecting a rise in the discrete-time systems. Very recently, in [24–26], the lower bound for $\mathcal{J}_D(\Delta x)$, was proposed in the manner of Wirtinger-based inequality. However, the lower bound for $\mathcal{J}_D(x) = \sum_{s=a}^{b} x^T(s)Mx(s)$ was not addressed in [24–26]. To obtain the less conservative conditions, the Lyapunov–Krasovskii functional has taken various augmented forms when it is constructed. For an example, the form $\sum_{s=a}^{b}\sum_{u=s}^{b}\begin{bmatrix} x(u) \\ \Delta x(u)\end{bmatrix}^T \mathcal{M} \begin{bmatrix} x(u) \\ \Delta x(u)\end{bmatrix}$ $(\mathcal{M} > 0)$ used in this work is one of Lyapunov–Krasovskii functional and its

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forward difference includes the summation term, $\sum_{s=a}^{b}\begin{bmatrix} x(s) \\ \Delta x(s) \end{bmatrix}^T \mathcal{M} \begin{bmatrix} x(s) \\ \Delta x(s) \end{bmatrix}$. At this point, for applying directly the results in [24–26] to various forms of Lyapunov–Krasovskii functional, we need the lower bound of $\mathcal{J}_D(x)$ as well as the one of $\mathcal{J}_D(\Delta x)$ because it contains the form $\mathcal{J}_D(x)$. In addition to this, the inequalities proposed in [24,25] were only presented like as the inequality of Wirtinger form. Therefore, to widen the usability of the inequality of Wirtinger form, there is a need to make its general form. For the details, its reasons will be explained in Remarks 1 and 2 of next section.

Motivated by the matters mentioned above, two stability conditions for discrete-time systems with time-varying delays will be proposed in this work. First of all, a new summation inequality is introduced as Proposition 1. Next, by utilizing the results of Proposition 1 and constructing the augmented Lyapunov–Krasovskii functional, a stability criterion for such systems is derived in Theorem 1 within the framework of LMIs [27]. Moreover, based on the result of Theorem 1, a stability criterion for discrete-time switched systems with time-varying delays is presented as Corollary 2. Here, switched systems consisted of a finite number of subsystems are a special sort of hybrid systems and have significant engineering background in the theoretical development as well as practical applications [28,29]. With this respect, the discrete-time systems with time-varying delays need to be analyzed for the stability problem of their switched model under arbitrary switching [30–33]. Various models such as linear and nonlinear continuous-time systems [34], time-varying systems [35] and nonlinear positive systems [36] were also studied in the research field of switched systems. Finally, through three numerical examples utilized in many previous works, the merit of the proposed summation inequality will be shown.

Notation: Throughout this paper, the used notations are standard. $\mathbb{R}$ and $\mathbb{N}^+$ stands for the sets of real numbers and positive integers, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional Euclidean space with the Euclidean vector norm $\|\cdot\|$ and the set of all $m\times n$ real matrices, respectively. $\mathbb{S}^n$ and $\mathbb{S}^n_+$ are the sets of symmetric and symmetric positive definite $n\times n$ matrices, respectively. $X > 0$ (<0) means symmetric positive (negative) definite matrix. $X^\perp$ denotes a basis for the null-space of X. $\mathtt{diag}\{\cdots\}$, $\mathtt{sym}\{X\}$, $\mathtt{col}\{x_1,\ldots,x_n\}$ and $\{y_i\}_{i=1}^n$ stand for, respectively, the (block) diagonal matrix, the sum of X and X^T, the column vector with the vectors $x_1,\ldots,x_2$, and the set of the elements $y_1,\ldots,y_n$. The symmetric terms in symmetric matrices and in quadratic forms will be denoted by $\star$. (This is used if necessary.) $X_{[f(t)]}$ means the sum of a constant matrix X_1 and a linear matrix $f(t)X_2$ for all real scalars $f(t)$; *i.e.*, $X_{[f(t)]} = X_1 + f(t)X_2$.

2. Problem formation and preliminaries

Consider the discrete-time switched systems with time-varying delays given by

$$\begin{aligned} x(k+1) &= A_\varsigma x(k) + A_{d\varsigma}x(k-h(k)), \\ x(k) &= \varphi(k), \quad k = -h_M, -h_M+1,\ldots,0, \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $\varphi(k)$ is an initial function. The delay function $h(k)$ is time-varying and satisfies $h_m \le h(k) \le h_M$, where h_m and h_M are known positive integers. ς meaning $\varsigma(k)$ for simplicity, is a piecewise constant function of time, called switching signal which takes its values in the finite set $\mathcal{S} = \{1,2,\ldots,N\}$, where $N \in \mathbb{N}^+$ is the number of subsystems. As in [28], let us assume that the switching signal ς is unknown a priori, but its instantaneous value is available in real time. $A_\varsigma, A_{d\varsigma} \in \mathbb{R}^{n\times n}$ are real known constant matrices for each $\varsigma \in \mathcal{S}$.

Remark 1. In practice, the delay bounds h_m and h_M dependent on the switching signal $\varsigma(k)$ can have different or same values. If we consider the switched systems with these delay bounds of different values, then the analysis for the systems would be very complex. In detail, let us define the delay $d(k)$ dependent on the switching signal $\varsigma(k)$ as $h_m(\varsigma(k)) \le d(k) \le h_M(\varsigma(k))$. Here, for simple analysis, it is assumed that the delay $h(k)$ is set in $\{\varsigma \in \mathcal{S}|[\min_\varsigma\{h_m(\varsigma(k))\}, \max_\varsigma\{h_M(\varsigma(k))\}]\}$. Then, each bound of $d(k)$ belongs to the bound of $h(k)$ even if the bounds of $d(k)$ are different. Therefore, in this work, the analysis results for the switched systems with $h(k)$ are guaranteed to the ones for the system with $d(k)$. At this time, in this work, it should be noted that $h_m = \min_\varsigma\{h_m(\varsigma(k))\}$ and $h_M = \max_\varsigma\{h_M(\varsigma(k))\}$.

The aim of this paper is to investigate the stability of the system (1) by applying a new summation inequality which will be introduced in the next section. The following fact and lemmas will be utilized in deriving main results.

Fact 1 (*[37]*)**.** Suppose $c \in \mathbb{R}$ is a scalar, and $\mathbf{g}_1$ and $\mathbf{g}_2$ in $\mathbb{R}^n$ are vectors. Then

$$\|c\mathbf{g}_1 + c\mathbf{g}_2\| \le |c|\,\|\mathbf{g}_1\| + |c|\,\|\mathbf{g}_2\|,$$

where $|c|$ denotes the absolute value of c.

Lemma 1 (*[14]*)**.** *For any vectors x_1 and x_2 in $\mathbb{R}^n$, matrices $R \in \mathbb{S}^n_+$, $M \in \mathbb{R}^{n\times n}$, nonnegative real scalars α_1 and α_2 satisfying $\alpha_1+\alpha_2 = 1$, and $x_i = 0$ if $\alpha_i = 0$ $(i = 1,2)$, the following inequality holds:*

$$\frac{1}{\alpha_1}x_1^TRx_1 + \frac{1}{\alpha_2}x_2^TRx_2 \ge \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \underbrace{\begin{bmatrix} R & M \\ \star & R \end{bmatrix}}_{\text{s.t. } >0} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$