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Extended hybrid model reference adaptive control of piecewise affine systems

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ABSTRACT

We discuss an extension to the adaptive control strategy presented in di Bernardo et al. (2013) able to counter eventual instabilities due to disturbances at the input of an otherwise $\mathcal{L}_2$ stable closed-loop system. These disturbances are due to the presence of affine terms in the plant and reference model. The existence of a common Lyapunov function for the linear part of the PWA reference model is used to prove global convergence of the error system, even in the presence of sliding solutions, as well as boundedness of all the adaptive gains.

1. Introduction

As notably highlighted in [1], adaptive control of switched systems is still an open problem. Recently, a novel model reference adaptive strategy has been presented in [2,3] that allows the control of multi-modal piecewise linear (PWL) plants. Specifically, a hybrid model reference adaptive strategy was proposed able to make a PWL plant track the states of an LTI or PWL reference model even if the plant and reference model do not switch synchronously between different configurations. While stability is guaranteed for PWL systems, for affine systems the presence of a non-square integrable disturbance term in the error equations and the possible occurrence of sliding solutions can render the proof of stability inadequate.

We wish to emphasize that the problem of large state excursions and instabilities caused by constant input disturbances on the closed loop system is a common problem of adaptive control systems seldom highlighted in the literature (see for example [4,5], and [6, Sec. 4.4.4 p. 173]). Indeed, adaptive systems can be represented as the negative feedback interconnection of a passive system (defined by the estimator) and a strictly positive real (SPR) transfer function. A simple application of the passivity theorem establishes that the overall system is $\mathcal{L}_2$-stable. However, this property does not ensure that the system will remain stable in the presence of external disturbances which are not $\mathcal{L}_2$.

The same problem is also true for recent extensions of MRAC schemes to switched systems including the hybrid extension of the Minimal Control Synthesis MRAC algorithm presented in [2]. The aim of this note is to present a modification of the control strategy presented in [2] able to guarantee asymptotic stability of the closed loop system even in the presence of sliding mode trajectories and bounded $\mathcal{L}_\infty$ perturbations due to the affine terms in the description of the plant and/or reference model. The idea is to add an extra switching action to the controller in [2] able to compensate the presence of such a disturbance. The proof of stability is obtained by defining an appropriate common Lyapunov function and analyzing its

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properties along the closed-loop system trajectories within each of the phase space regions where the plant and reference model are characterized by different modes, and along their boundaries. We show that, even in the presence of sliding mode trajectories, the origin of the closed-loop error system is rendered asymptotically stable by the extended strategy presented in this paper. A preliminary version of the algorithm suitable to control bimodal piecewise affine system can be found in [7,8], while experimental validation results are reported in [9]. A possible extension to discrete-time piecewise-affine (PWA) plants of the approach can be found in [10].

2. Problem statement and definitions

Assume that the state space $\mathbb{R}^n$ is partitioned by some smooth boundaries into M non-overlapping domains, say $\{\Omega_i\}_{i\in\mathcal{M}}$ with $\mathcal{M} = \{0, 1, \dots, M-1\}$ such that $\bigcup_{i=0}^{M-1} \Omega_i = \mathbb{R}^n$ and that, given any two generic indexes i_1 and $i_2 \in \mathcal{M}$ (with $i_1 \neq i_2$), $\Omega_{i_1} \cap \Omega_{i_2} = \emptyset$.

Let the plant be described by an n-dimensional multi-modal PWA system whose dynamics are given by:

$$\dot{x} = A_i x + Bu + B_i \quad \text{if } x \in \Omega_i,\ i \in \mathcal{M}, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the scalar input, and the matrices A_i, B, B_i $(i = 0, 1, \dots, M-1)$ are assumed to be in control canonical form, i.e.

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & & 1 \\ a_i^{(1)} & a_i^{(2)} & \cdots & a_i^{(n)} \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b \end{bmatrix}, \qquad B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b_i \end{bmatrix}, \tag{2}$$

with $b > 0$. Note that all entries on the last row of the plant matrices A_i, B and B_i are supposed to be unknown. Also, notice that if the plant is not into canonical form, a transformation presented by the authors in [11] can be used, under certain assumptions, as part of the design process to recast the plant in the required form. In many cases, though, electromechanical systems modeled via a Lagrangian approach are structurally already into the required canonical form.

The problem we wish to solve is to find an adaptive piecewise feedback law $u(t)$ to ensure that the state variables of the plant track asymptotically the states, say $\widehat{x}(t)$, of a reference model independently from their initial conditions.

Here, we assume that the reference model can be either an LTI system, or a multi-modal PWA system:

$$\dot{\widehat{x}} = \widehat{A}_{\widehat{i}}\widehat{x} + \widehat{B}r + \widehat{B}_{\widehat{i}} \quad \text{if } \widehat{x} \in \widehat{\Omega}_{\widehat{i}},\ \widehat{i} \in \widehat{\mathcal{M}}, \tag{3}$$

where the state $\widehat{x} \in \mathbb{R}^n$, $\widehat{\mathcal{M}} \triangleq \{0, 1, \dots, \widehat{M}-1\}$, $\{\widehat{\Omega}_{\widehat{i}}\}_{\widehat{i}\in\widehat{\mathcal{M}}}$ is a partition of $\mathbb{R}^n$ into $\widehat{M}$ domains obtained by some smooth boundaries and $r \in \mathbb{R}$ is the input to the reference model. Note that the reference model may possess a number of modes different from the one of the plant, $\widehat{M} \neq M$. Furthermore, we assume that the reference model defined as in (3) is chosen so as not to exhibit sliding solutions and that it is well-posed given the initial condition $\widehat{x}(0) = \widehat{x}_0$. In many practical cases, the aim of the control action can be that of compensating the discontinuous nature of the plant. In these situations, the control design presented above offers a simple and viable solution for this to be achieved by simply choosing a smooth or smoother reference model. This often corresponds to the conventional choice of an asymptotically stable LTI reference model in the case of smooth systems.

As for the plant, the matrices of the reference model are chosen to be in the companion form given by ($\widehat{i} = 0, 1, \dots, \widehat{M}-1$):

$$\widehat{A}_{\widehat{i}} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & & 1 \\ \widehat{a}_{\widehat{i}}^{(1)} & \widehat{a}_{\widehat{i}}^{(2)} & \cdots & \widehat{a}_{\widehat{i}}^{(n)} \end{bmatrix}, \qquad \widehat{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \widehat{b} \end{bmatrix}, \qquad \widehat{B}_{\widehat{i}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \widehat{b}_{\widehat{i}} \end{bmatrix} \tag{4}$$

with $\widehat{b} > 0$.

In what follows, we use the standard notation in [12] (also adopted in [13]), for both the switching instants of the plant and reference model. More precisely, the switching sequence of the plant is given by:

$$\Sigma = \{x_0, (i_0, t_0), (i_1, t_1), (i_2, t_2) \dots (i_p, t_p) \dots \mid i_p \in \mathcal{M}, p \in \mathbb{N}\}, \tag{5}$$

where $t_0 = 0$ is the initial time instant and x_0 is the initial state. Note that, as in [12], when $t \in [t_p\ ; t_{p+1})$, $x(t)$ belongs to Ω_{i_p} by definition and, thus, the i_pth subsystem is active. Obviously, the switching sequence Σ may be finite or infinite. If there is a finite number of switchings, say p, then we set $t_{p+1} = \infty$.