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Ultimate bound minimisation by state feedback in discrete-time switched linear systems under arbitrary switching

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ABSTRACT

We present a novel state feedback design method for perturbed discrete-time switched linear systems. The method aims at achieving (a) closed-loop stability under arbitrary switching and (b) minimisation of ultimate bounds for specific state components. Objective (a) is achieved by computing state feedback matrices so that the closed-loop subsystem evolution matrices generate a solvable Lie algebra (namely, they are all upper triangular in a common coordinate basis). Previous results derived an iterative algorithm that computes the required feedback matrices, and established conditions under which this procedure is possible. Based on these conditions, objective (b) is achieved by exploiting available degrees of freedom in the iterative algorithm.

1. Introduction

In the last decade there has been increasing research activities in the areas of stability and stabilisability of switched systems; see, for example, [1–3]. An example of switched systems is a system with time-varying dynamics, which switches within a known set of modes, or subsystems, indexed by a switching signal. A problem of interest is that of stability under arbitrary switching, which consists in obtaining conditions that guarantee stability of the switched system for every switching signal. Finding these conditions in general is not a simple task except for special cases, such as when the subsystems are pairwise commutative, symmetric or normal [1, Chapter 2]. A well-known necessary and sufficient condition for exponential stability under arbitrary switching is the existence of a common Lyapunov function (CLF) for all

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subsystems [4]. As a CLF might be rather complex and difficult to find, most developed work focuses on the existence of a common quadratic Lyapunov function (CQLF).

While most efforts on stability and stabilisation of switched systems deal with asymptotic stability of the origin (as equilibrium point of the system), it might not be possible to achieve asymptotic stability in some situations, such as when the switched system is subject to non-vanishing disturbances. In these situations, one may seek practical stability of the system, in the sense that the system trajectories are required to ultimately lie inside a bounded region around the origin. The *ultimate bounds* of the states of the system characterise such region, which should be sufficiently small for good asymptotic disturbance attenuation performance.

Estimates of the state ultimate bounds can be obtained through the use of level sets of suitable Lyapunov functions (see for example [5, Section 9.2]). This approach is applicable to a general class of nonlinear systems, but may produce overly conservative bounds in linear systems, since the structure of the system is generally lost in the Lyapunov function [6]. Tighter estimates of ultimate bounds in linear systems can be obtained through a *componentwise* analysis technique proposed in [6,7], which preserves system structure and dispenses with Lyapunov functions.

The problem of reducing the effect of disturbances by feedback has been long studied and a number of robust control methods exist for the minimisation of ultimate bounds in linear systems (e.g., [8–10]), although typically relying on Lyapunov analysis and induced norms (such as l_2 and l_∞). Compared to the componentwise approach proposed in this paper, induced norm minimisation approaches may be conservative in the sense that they might not yield the best results for some specific state components representing a meaningful or a physical quantity. An example projecting this idea is studied in [11]. The minimisation of componentwise ultimate bounds by feedback design has been studied in [12] for linear time-invariant (LTI) systems. The authors in [12] have shown that arbitrarily small ultimate bounds can be guaranteed in continuous-time systems by assigning closed-loop eigenvalues with arbitrarily large negative real part when disturbances are "matched" to the control input (that is, disturbances in the span of the system's control input matrix). For discrete-time systems, however, there is a fundamental limitation in rendering these ultimate bounds arbitrarily small, depending on the way the disturbance affects the state equations. In this regard, the problem of ultimate bound minimisation for discrete-time LTI systems has recently been studied in [13], where conditions were derived so that the ultimate bound on one (or more) state components can be minimised to its least possible value via eigenvalue–eigenvector assignment.

Ultimate boundedness of switched systems subject to uncertainty and disturbances has been the focus of attention recently. Necessary and sufficient conditions were derived in [14] for autonomous switched linear systems to have a finite disturbance attenuation level under arbitrary switching. The authors also provide sufficient conditions under which disturbance attenuation can be attained under a dwell-time switching constraint. Similar results with dwell-time switching constraints have been obtained for switched Euler–Lagrange systems in [15]. In [16], the authors present sufficient conditions on the existence of a CLF for a continuous-time switched linear system subject to parameter uncertainties to achieve uniformly ultimate boundedness under arbitrary switching. The stability and componentwise state ultimate bounds of autonomous switched systems under arbitrary switching have been analysed in [17–19], where an iterative algorithm that derives a CQLF is proposed.

The present paper examines the problem of feedback design for practical stabilisation with ultimate bound minimisation. An iterative algorithm is proposed to obtain the smallest possible bounds for specific state components in discrete-time switched linear systems under arbitrary switching. The methodology extends an algorithm from [20], which iteratively seeks a set of stabilising state feedback gains that render the closed-loop subsystem matrices simultaneously upper-triangular after a change of coordinates common to all subsystems. This closed-loop upper-triangular structure is a desirable property, since then the stable closed-loop subsystem matrices will generate a solvable Lie-algebra, which guarantees the existence of a CQLF [21].

The results in this paper improve on existing results from [20], which address stabilisation of switched linear systems under arbitrary switching. The work in [20] is one of the few available works on feedback control design in the switched system context [22,23]. The present paper deals with discrete-time switched linear systems in the presence of non-vanishing bounded disturbances, in contrast with [20], where no disturbance affects the system. The algorithm from [20] is modified in the current paper by imposing additional structure to the closed-loop subsystems to achieve disturbance attenuation by minimising componentwise state ultimate bounds.

The first contribution of this paper is to derive conditions in terms of eigenstructure of the perturbed switched system in order for the trajectories of one or more components of the state to lie within the smallest possible bound in at most one time step. Next, we extend the results of [20] by exploiting the available degrees of freedom in the iterative triangularisation algorithm by imposing a set of conditions on the common eigenvectors at each iteration of the algorithm. The main contribution of the paper is an eigenstructure assignment procedure embedded in the extended version of the aforementioned algorithm such that the resulting stabilising feedback laws achieve the minimum possible ultimate bound for one or more states of the switched system under arbitrary switching. The results in this paper build upon preliminary work communicated in the conference paper [24].

The layout of the remainder of the paper is as follows. In Section 2, structural conditions on the system matrices for one or more ultimate bound components to be the minimum possible are presented. In Section 3, ultimate bound minimisation is addressed through iterative eigenstructure assignment, which is performed via modifications in the iterative algorithm of [20]. In Section 4, a numerical example shows the effectiveness of the proposed algorithm and finally, Section 5 concludes the paper.