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An implicit systems characterization of a class of impulsive linear switched control processes. Part 2: Control

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ABSTRACT

Our paper focuses on a fundamental structural property associated with a family of linear switched control systems in the presence of impulsive dynamics. We consider dynamic processes governed by piecewise linear ODEs with controlled location transitions. Using the newly developed implicit systems characterization technique, we rewrite the initially given impulsive switched system as an implicit dynamic model. This auxiliary representation of the original control system makes it possible to hide the switched phenomenon and to apply the conventional approach to the resulting implicit dynamic model. The proposed algebraic-based modeling framework follows the celebrated behavioral approach (see Polderman and Willems, 1998) and makes it possible to apply to the switched dynamics some classic techniques from the well-established time-invariant implicit systems theory. The analytic results of our paper constitute a formal theoretical extension of switched control systems methodology and can be used (as an auxiliary step) in a concrete control design procedure. The second part of our manuscript is devoted to control aspects.

1. Introduction

This paper discusses a behavioral based analytic approach to a class of impulsive linear switched dynamic systems with controlled location transitions (see *e.g.*, [1] and the references therein). The main aim of our contribution is to show that a constructive analysis of a certain class of impulsive linear switched systems with controllable location transitions can be incorporated into the classic behavioral concept. This "embedding" of the linear switched systems into the wide class of implicit dynamic models involves the application of basic facts from the implicit systems theory to the constructive analysis of the initially given sophisticated switched systems.

In Part I of our manuscript (see [2]) we have shown how the linear time-invariant implicit systems theory can be efficiently used for adequate modeling of certain classes of switched systems with autonomous location transitions, Σ_{sw}. In fact, we have established a new modeling possibility for a class of switched systems using an auxiliary linear time invariant *rectangular implicit representation* that describes the so-called common fixed structure and a specific set of *algebraic constraints*. The significance of the results presented in [2] can be summarized as follow: the set of all admissible solutions of Σ_{sw} is included into the solution set of an auxiliary linear time-invariant *implicit rectangular representation* $\mathfrak{R}^{ir}$. Thus, *if*

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we determine a linear time-invariant control design for $\mathfrak{R}^{ir}$, we also can apply the same control strategy to Σ_{sw}. The results of [21] and the corresponding control scheme are in fact based on a PD-type control design procedure. In that case the switching effects are no longer present in the resulting system (they are unobservable). We also refer to [3] for an effective approximation technique associated with the above-mentioned feedback control algorithm. In this paper, we clarify the reachability and pole assignment questions for this kind of dynamic systems. We also show that the embedding technique mentioned above (Σ_{sw} into $\mathfrak{R}^{ir}$) makes it possible to design the stabilizing control strategies using the usual pole placement technique (see e.g., Σ_{sw}).

The paper is organized as follows. Section 2.1 summaries some necessary results from the first part of the manuscript, namely, from [2]. Section 2.2 is devoted to the basic geometric conditions that guarantee the existence of a proportional and derivative feedback control associated with the set of complex eigenvalues under consideration. In Section 3 we develop stability conditions for the whole set of implicit global representations we study. The examined class of switched systems is an application result of the proposed proportional and derivative feedback control design. In Section 4 we propose a proportional and derivative feedback control law that makes it possible to treat the internal variable structure. Section 5 summarizes our contribution. All the necessary technical proofs can be found in Appendices.

2. Preliminaries

2.1. Implicit representation of non-stationary switched systems

In [2], we have studied the implicit representation of the class of linear switched system (LSS) [1] described by the following space representations $\mathfrak{R}^{ss}(A_q, B, C_q)$:

$$\frac{\mathrm{d}}{\mathrm{dt}}\bar{x} = A_q\bar{x} + Bu \quad \text{and} \quad y = C_q\bar{x}; \quad t \in (T_{i-1}, T_i)\,, \lim_{t\to T_{i-1}^+} x(t) = c_i, \quad \forall i \in \mathbb{N}, \tag{2.1}$$

where $0 = T_0 < T_1 < \cdots < T_{i-1} < T_i \ldots, i \in \mathbb{N}, \lim_{i\to\infty} T_i = \infty$ is a given time partition and $\bar{x} \in \mathbb{R}^{\bar{n}}$ denotes the system state. We also have the following notation $u \in \mathbb{R}^m$ for the system input and $y \in \mathbb{R}^p$ is the corresponding output. By $c_i \in \mathbb{R}^{\bar{n}}$ we denote here the initial conditions associated with a switching time T_{i-1}, $i \in \mathbb{N}$. The q are elements of a finite set of indices (locations) $\mathcal{Q}$ such that for each $[T_{i-1}, T_i)$ the system remains in a location $q \in \mathcal{Q}$ according to $\mathfrak{s} : \{[T_{i-1}, T_i) \subset \mathbb{R}^+, i \in \mathbb{N}\} \to \mathcal{Q}$, $\mathfrak{s}([T_{i-1}, T_i)) = q$. We have assumed that the systems matrices A_q and C_q associated with the LSS under consideration have a specific structure[1]

$$A_q = \overline{A}_0 + \overline{A}_1\overline{D}(q) \quad \text{and} \quad C_q = \overline{C}_0 + \overline{C}_1\overline{D}(q), \tag{2.2}$$

where $\overline{A}_0 \in \mathbb{R}^{\bar{n}\times\bar{n}}$, $\overline{C}_0 \in \mathbb{R}^{p\times\bar{n}}$, $\overline{A}_1 \in \mathbb{R}^{\bar{n}\times\hat{n}}$, $\overline{C}_1 \in \mathbb{R}^{p\times\hat{n}}$, and $\overline{D}(q) \in \mathbb{R}^{\hat{n}\times\bar{n}}$. We assume that rank $B = m$, rank $C_q = p$, rank $\overline{C}_1 = p$ and that $\overline{D}(q)$ varies linearly with respect to q, namely

H 1. $\ker B = \{0\}$ and $\operatorname{Im} C = \mathbb{R}^p$.

H 2. Given two fixed locations $q_a, q_b \in \mathcal{Q}$ and a scalar $\alpha \in \mathbb{R}$, we obtain $\overline{D}(q_a + \alpha q_b) = \overline{D}(q_a) + \alpha\overline{D}(q_b)$.

Example 1. We now consider (2.1)–(2.2) with the following matrices

$$\begin{array}{l} \overline{A}_0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \overline{A}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ \overline{C}_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}, \qquad \overline{C}_1 = -1, \qquad \overline{D}(q) = \begin{bmatrix} q_1 & q_2 \end{bmatrix}, \\ q = (q_1, q_2) \in \mathcal{Q} = \{q_a, q_b, q_c, q_d\}, \\ q_a = (-1, -1), \qquad q_b = (-1, 0), \qquad q_c = (-1, -5), \qquad q_d = (1, 1). \end{array} \tag{2.3}$$

For a given pair (q_1, q_2) the corresponding transfer function can be expressed in the form

$$F_q(\mathrm{s}) = C_q(\mathrm{sI} - A_q)^{-1}B = \frac{-(q_2\mathrm{s} + q_1)}{\mathrm{s}^2 - (q_1 + q_2)\mathrm{s} - (1 + q_1 + q_2)}\,. \tag{2.4}$$

Consequently, we have $F_{q_a}(\mathrm{s}) = 1/(\mathrm{s}+1)$, $F_{q_b}(\mathrm{s}) = 1\big/\big(\mathrm{s}(\mathrm{s}+1)\big)$, $F_{q_c}(\mathrm{s}) = (5\mathrm{s}+1)\big/\big((\mathrm{s}+1)(\mathrm{s}+5)\big)$ and $F_{q_d}(\mathrm{s}) = -1/(\mathrm{s}-3)$.

[1] Let us discuss shortly the main motivation of the structure we study. Consider the following system: $\mathrm{d}x/\mathrm{d}t = \overline{A}_0x + \overline{A}_1v + Bu$, $y = \overline{C}_0x + \overline{C}_1v$, where v is an extra control input related to the switched-type state feedback controller $v = \overline{D}(q)x$.