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Robust stabilization of a class of state-dependent jump linear systems[☆]

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ABSTRACT

In this article, we consider a continuous-time state-dependent jump linear system (SDJLS), a kind of stochastic hybrid system, with the presence of uncertainties in system parameters. In SDJLS, we consider that the transition rates of the underlying random jump process depend on the state variable. In particular, we assume the transition rates to have different values across suitably defined sets to which the state of the system belongs, and address a problem of robust stability and stabilization analysis. We obtain sufficient conditions for robust stability and state-feedback stabilization in terms of linear matrix inequalities (LMIs). We validate the obtained sufficient robust stability and stabilization conditions with numerical examples.

1. Introduction

Many dynamical systems subject to random abrupt changes in their structure and parameters can be adequately modeled by random jump linear systems (RJLSs). RJLSs are a particular class of stochastic switching systems, which consists of a set of linear systems, also called modes, and switching among them is governed by a random jump process. It finds several applications such as manufacturing systems, networked control systems, economics, aircraft systems, see for example [2–5], etc.

When the random jump process of RJLS is assumed to be a finite state time-homogeneous Markovian process (or chain) with known transition rates (or probabilities), then this particular class of systems are widely known as Markov jump linear systems (MJLSs) in the literature. Many important results related to stability and control design of MJLSs can be found for instance in [6–10] and the references therein. In general, the studies of MJLSs assume that the underlying random jump process is time-homogeneous Markov that imply time-invariant transition rates (or probabilities), which is quite a restrictive assumption.

In this article, we consider the analysis of RJLS when the random jump process depends on the state variable. Such a class of systems are referred to as "state-dependent jump linear systems (SDJLSs)" in this article. The SDJLS modeling of dynamical systems is motivated by following scenarios. In the analysis of random breakdown of components, the age, wear, and accumulated stress of a component affect its failure rate, for instance. Thus, it is reasonable to assume that the failure

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rate of a component is dependent on state of the component at age t [11], where the state variable may be an amount of wear, stress, etc. Also, for instance, in [12], a state-dependent Markov process was used to describe the random breakdown of cylinder lines in a heavy-duty marine diesel engine. Also, a state-dependent regime switching model was considered in [13] to model financial time series. Consider a case of stock market with two regimes: up and down. The transition rates between the regimes usually result from the state of economy, the general mood of the investors in the market, etc., which can be regarded in general as the state of the market. As an another scenario, consider a distributed vehicle network with bounded transmission information delays, where the delays can vary depending on the distance among the vehicles that is part of the state of the system. One may find more examples or scenarios of this kind in the literature.

So far, only a few works addressed the issues of stability and control of SDJLSs. A study of hybrid switching diffusion processes, a kind of continuous state-dependent jump non-linear systems with diffusion, has been carried out by [14] by treating ergodic properties, existence, uniqueness, stability of the solutions, etc. In [15], the authors considered that the transition rates of the random jump process depend on both the state variable and the control input, which affect the time scale of the random jump process, thus affecting its transition rate, and obtained a control policy for a given functional using stochastic maximum principle. In [16], a two-time scale model of production plant was regarded as a jump diffusion model with state-dependent failure rates, and obtained an optimal control.

In this article, we consider SDJLS with uncertainties in system parameters, and address robust stability and stabilization problems. Robust stability problem for RJLSs has been addressed by [17–20] and references therein. To our knowledge, a robust stabilization of SDJLS has not been addressed in the literature.

In this article, we consider the state-dependent jump process as follows. We consider that the state space is partitioned into disjoint sets and assume different transition rate matrices of jump process across different sets. It is a reasonable assumption because the state of the system at any time belong to one of the partitioned sets, and the transition rates can be considered to have different values across the predefined sets. By using Dynkin's formula, we obtain numerically tractable sufficient conditions for robust stability and stabilization in terms of linear matrix inequalities (LMIs).

The remainder of this article is organized as follows. Section 2 introduces a mathematical model of SDJLS studied in this article and the problem formulation. In Section 3, we provide preliminary results. In Section 4, we give sufficient conditions for robust stability and stabilization of SDJLS. Section 5 presents two numerical examples to illustrate the proposed results. In Section 6, we give possible future directions of research and Section 7 concludes the paper. Finally, we give some of the proofs in the Appendix to improve readability of the article.

Notation: Let $\mathbb{R}^n$ be the n-dimensional real Euclidean space where $\|.\|$ denotes the standard vector norm and for a matrix A, $\|A\|$ denotes the corresponding induced matrix norm of A. For a matrix A, A^T denotes the transpose, $\lambda_{\min}(A)(\lambda_{\min}(A))$ denotes the minimum (maximum) eigenvalue of A. Let $\mathbb{I}_n$ denotes an identity matrix of dimension $n \times n$ and $\mathbb{I}$ an identity matrix of appropriate dimension. Given a real symmetric matrix M, $M \prec 0$ ($M \preceq 0$) and $M \succ 0$ ($M \succeq 0$) denote that the matrix M is negative definite (negative semi-definite) and positive definite (positive semi-definite) respectively. Symmetric terms in block matrices are denoted by $\star$. The diagonal matrix formed from its vector arguments is denoted by diag$\{.\}$. Let $\mathbb{N}_{\geq 0}$ be the set of natural numbers including 0. The empty set is represented by ϕ. For any $a, b \in \mathbb{R}$, $a \wedge b$ represents the minimum of two numbers a, b. $I_A(x)$ is the standard indicator function which has a value 1 if $x \in A$, otherwise has a value 0; in the absence of the argument x, let I_A denote an indicator function which has a value 1 if A is true otherwise has a value 0. The mathematical expectation of a random variable X is denoted by $\mathbb{E}[X]$. Let $g(X_t)$ be an arbitrary functional of a stochastic process X_t, then $\mathbb{E}[g(X_t)]_{X_t=X}$ denotes the expectation of the functional $g(X_t)$ at $X_t = X$. Further notation will be introduced as when required.

2. Problem description

Consider a SDJLS with norm bounded uncertainties in a fixed probability space $(\Omega, \mathcal{F}, \Pr)$:

$$\begin{cases} \dot{x}(t) = \left(A_{\theta(t)} + E_{\theta(t)}\Psi(t)F_{\theta(t)}\right)x(t) + \left(B_{\theta(t)} + E_{\theta(t)}\Psi(t)FG_{\theta(t)}\right)u(t), \\ x(0) = x_0, \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $x_0 \in \mathbb{R}^n$ is the initial state, $A_{\theta(t)}$, $B_{\theta(t)}$, $E_{\theta(t)}$, $F_{\theta(t)}$ and $G_{\theta(t)}$ be system matrices of appropriate dimensions, which depend on $\theta(t)$ and are assumed to be known. Let $\Psi(t)$, denoting uncertainties in system (1), be an unknown real matrix that satisfies $\Psi^T(t)\Psi(t) \preceq \mathbb{I}$. Let $\{\theta(t),\ t \geq 0\} \in S := \{1, 2, \dots, N\}$, describing the mode process of the system, be a finite space continuous time state-dependent jump process. In this article, we consider characterization of state-dependent jump process as follows: we partition the state space into finite disjoint subsets and then consider different transition rates across these sets. In the sequel, we provide a partitioning of the state space.

Assumption 1. Let $\mathcal{K} \triangleq \{1, 2, \dots, K\}$. We assume that $\mathcal{C}_1, \mathcal{C}_2, \dots \mathcal{C}_K \subseteq \mathbb{R}^n$ are nonempty Borel sets in $\mathbb{R}^n$ and also disjoint, i.e., $\cup_{\iota=1}^{K} \mathcal{C}_\iota = \mathbb{R}^n$ and $\mathcal{C}_p \cap \mathcal{C}_q = \phi$ for any $p, q \in \mathcal{K},\ p \neq q$. We further assume that each of the sets $\mathcal{C}_m$ for $m \in \mathcal{K}$ is a connected set.