Nonlinear Analysis: Hybrid Systems 18 (2015) 72–84

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Practical stability of switched uncertain nonlinear systems using state-dependent switching laws

Angelica Mendoza-Torres [a], Ilse Cervantes [b,*,1], Hugo Cabrera-Ibarra [b]

[a] Metallurgy Institute, UASLP, Av. Sierra Leona 550, Col. Lomas 2a. Seccion, 78210, San Luis Potosi, S.L.P., Mexico

[b] Applied Mathematics Division, IPICyT, Camino a la Presa San Jose 2055, Col. Lomas 4a. Seccion, 78216, San Luis Potosi, S.L.P., Mexico

ARTICLE INFO

Article history:
Received 27 September 2014
Accepted 2 June 2015

Keywords:
Practical stability
Restricted switching
Switching law
Robust stability
Switched impulsive systems

ABSTRACT

In this paper, it is proven that a state dependent switching law designed for integrator switched systems can be used to stabilize complex switched uncertain nonlinear systems as well as a class of impulsive systems, provided that the upper bound of the uncertainty terms satisfy a relationship with the nominal system parameters estimation of the nominal system parameters. In order to establish such a result, firstly, a switching law is proposed to ε-practically stabilize an uncertain integrator switched system; secondly, the proposed switching law is used to prove that the trajectories of a general class of uncertain nonlinear systems are ε-practical stable and connections to switched nonlinear impulsive systems are revealed. Numerical simulations allow us to illustrate the proposed stability results and an example in a power electronics device is used to show the applicability of the proposed state-dependent switching control strategy.

1. Introduction

Studying issues such as controllability, stabilizability and stabilization of impulsive and switched systems has gained a lot of attention lately, due to the high relevance of these systems in engineering. Given the great diversity of existing switched and impulsive systems, it is possible to find in the literature a corresponding variety of results regarding stability, most of them focused on one of the following two problems: Stability with restricted switching [1–3], and stability with arbitrary switching [4–9].

If the structure of every subsystem (mode) cannot be modified using feedback, the only degree of freedom available to ensure stability is the switching action. The problem of finding switching laws that ensure that a system becomes stable is known as the *switched stabilizability problem* and, in general, it can be solved using restricted switching.

The derivation of stabilizing switching laws can be performed by one of the following approaches:

1. Formulating a stabilization problem, generally using Lyapunov or Lyapunov-like functions for the stability analysis (see [10–12] and references therein) or
2. Using the stabilizability conditions of the switched system to derive directly the switching law (see [13–17]).

The first approach has been widely adopted and depending on the modes of the switched system (all stable, all unstable or some unstable), the problem can be solved using suitable dwell times for the unstable modes, resulting in a time-dependent

* Corresponding author. Tel.: +52 444 8342000x7221.
E-mail addresses: ruby.angelica@gmail.com (A. Mendoza-Torres), ilse@ipicyt.edu.mx (I. Cervantes), cabrera.ipicyt@gmail.com (H. Cabrera-Ibarra).

[1] SEPI ESIME-CU IPN.

http://dx.doi.org/10.1016/j.nahs.2015.06.002
1751-570X/

switching law ([12,18–20], see also [21–33]). It has been shown that if all the subsystems are stable or there exists a stable convex combination of the modes, it is possible to derive state-dependent switching laws (see [34–37]). However, a drawback of most of the existing proposals based on Lyapunov stability analysis is that the modes should share a common equilibrium point.

On the other hand, the second point of view uses the controllability (or stabilizability) conditions to derive directly the switching law (see [13–17]); therefore the results are not restricted to systems that share a common equilibrium point, and they can be applied regardless the stability properties of the modes. This point of view generally leads to state-dependent switching laws, the contribution of this paper use this methodology.

State-dependent switching laws have been proposed in [11,15,17,32,34–36]. Specifically, in [34] the switched stabilizability problem is analyzed for a pair of unstable linear systems, therein a switching strategy is proposed using Lyapunov functions and a stable convex combination of the system matrices. These results have been recently extended in [36] for switched linear systems with unstable multiple subsystems. On the other hand, in [35] a state-dependent switching strategy that uses the smallest projection of the vector field of the active mode is proposed to stabilize a class of switched nonlinear systems. In [32] the stabilization of discrete-time switched linear systems is studied. A state-feedback path-wise switching law is proposed and it is proven that such law is universal in the sense that any stabilizable switched linear system admits a stabilizing switching law in this set. In [32] the authors also propose an algorithm to compute such switching law.

From a practical viewpoint, it is important to study the effect of system uncertainty in the design of switching laws. There are significant results in the literature concerning switched uncertain systems such as [29,30]. In [29], it is used a polytopic (linear) system description to derive stability conditions under uncertainty, the derivation of the switching law is performed using composite quadratic functions and the directional derivative along a sliding mode. While in [38] a hysteresis-based switching logic is presented, which depends on both time and state. This approach is used to switch among a family of controllers, whose objective is to regulate a linear system with unknown parameters taking values in infinite sets. In spite of these contributions, it is still unclear, the effect of the uncertainty of the switching law on the system stability.

This paper is focused on the design of a state-dependent switching law for nonlinear uncertain systems, departing from the stabilizability conditions for switched systems, such departing point allows us to propose a switching law that stabilizes different type of systems. In particular we focus on the switched stabilizability problem for nonlinear uncertain systems using state-dependent functions. Our goal is to show that a simple state-dependent switching law is able to stabilize switched uncertain nonlinear systems of arbitrary dimension even if their subsystems are stable or unstable, or do not share equilibrium points. Moreover, we prove that the use of a switching law in switched uncertain nonlinear system, leads us to notice the existence of an associated switched impulsive system that can also be stabilized using the same switching law.

With respect to existing works, our contribution relies on the fact that we derive stabilizing state-dependent switching laws for systems that are (i) uncertain and nonlinear of arbitrary dimension, (ii) their subsystems do not have or do not share equilibrium points which, to the authors knowledge, constitute a problem that has not been addressed before. Moreover, we reveal connections between the stabilization of switched systems and switched impulsive systems.

Even if in [14] a state-dependent switching law is provided for systems that do not share equilibrium points, the authors do not analyze the stability of such law. In this paper, we show that the dwell times produced by the time-dependent switching law in [14] are different and larger than those derived by the proposed state-dependent switching law. Moreover in [14], the case with uncertainty is not analyzed and a direct extension of the switching law in [14] to the case of uncertain systems would lead to an uncertain and unverifiable switching law. Moreover, impulsive systems are not analyzed in [14]. On the other hand, our contribution with respect to our previous work [39] is to study the stabilization of uncertain system with state-dependent switching laws and to reveal connections with switched impulsive systems.

This paper is organized as follows: some definitions and the problem formulation are given in Section 2. Stability results and connections with impulsive systems are presented in Section 3. Numerical simulations as well as a real application in power electronics are used to illustrate our findings in Section 4 and finally in Section 5, some conclusions are given.

2. Preliminaries

In this section, basic definitions and remarks to clarify the main contribution of this paper are introduced. For convenience, and without loss of generality, all definitions and results are stated about the origin of $\mathbb{R}^n$. Other operating points can be obtained with an appropriate shifting change of variables.

Let us consider the switched uncertain nonlinear time-invariant system:

$$\dot{x} = f_i(x) + \Delta f_i(x) \quad i \in I = \{1, \dots, M\}, \tag{1}$$

where $\Delta f_i(x)$ represents the uncertainty that includes not only parametric but also structural uncertainty, $\Delta f_i(x), f_i : \mathbb{R}^n \to \mathbb{R}^n$ are locally Lipschitz continuous functions. Every subsystem i may be stable and/or unstable, and they may or may not have a common equilibrium point. The election of the active subsystem is performed by a switching law S, which is a function $S : \mathbb{R}^n \times \mathbb{R} \longrightarrow \sum_{[t_0,t_f)}$, where $\sum_{[t_0,t_f)}$ is the set of all possible switching sequences σ in $[t_0, t_f)$ for a given initial condition (x_0, t_0) defined as:

$$\sigma(x_0, t_0) = \sigma = \{(i_1, \theta_1), (i_2, \theta_2), \dots, (i_K, \theta_K)\}, \tag{2}$$