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Stability analysis of positive switched systems via joint linear copositive Lyapunov functions

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ABSTRACT

In this paper, we study the asymptotic stability of continuous-time positive switched linear systems for the case when each subsystem is only stable. By using the so-called "joint linear copositive Lyapunov function" (JLCLF) generalizing the common linear copositive Lyapunov function, we show that the system remains asymptotically stable under appropriate switching if it has a JLCLF. Then, the main result is extended to positive switched linear systems with time delay.

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1. Introduction

A switched system is a hybrid dynamical system consisting of a family of (either discrete-time or continuous time) subsystems and a rule that regulates the switching among them. Up until now, the stability theory of switched systems has attracted the attention of multi-disciplinary researchers in a wide range [1,2]. If the variables involved in the system description are constrained by their physical nature to take positive values (e.g. they may represent concentrations, probabilities, population levels, temperatures, etc.), then we have positive switched systems which consist of a family of positive subsystems [3,4] and a rule to switch among them. In the past decades, positive switched system has been studied extensively due to its broad applications in many areas such as communication systems [5], formation flying [6], and systems theory [7,8].

As one of the most fundamental problems for positive switched systems, the stability has been highlighted by many researchers in recent years. A mass of literature is focused on the asymptotic stability of positive switched linear systems under arbitrary switching. Such a problem is usually studied by using a common linear copositive Lyapunov function (CLCLF) [9–14]. By considering the CLCLF of the dual system, the asymptotic stability under arbitrary switching was investigated for the case of unbounded time-varying delay in [15,16]. A switched linear copositive Lyapunov function was proposed in [17] to study the asymptotic stability of discrete-time positive switched linear systems under arbitrary switching. Another issue for positive switched system is the stabilization problem. It was shown in [18] that if a discrete-time positive switched linear system is stabilizable, it can be stabilized by means of a periodic switching sequence. Exponential stabilizability of continuous-time positive switched linear systems was studied in [19] via control copositive Lyapunov functions. The stabilization based on average dwell time switching was discussed for both continuous-time and discrete-time positive switched systems in [20–23]. Very recently, copositive polynomial Lyapunov function was proposed in [24], which generalizes some existing copositive types of Lyapunov functions.

Unlike most of references, we here focus on the asymptotic stability of continuous-time positive switched linear system for the case when each subsystem is only stable. To investigate this problem, we introduce a class of joint linear copositive Lyapunov functions (JLCLFs) which comprise CLCLFs as special cases. Appropriate switching signals have been designed to

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guarantee the asymptotic stability of the system if it has a JLCLF. We also extend the main result to the case of time delay. Finally, two numerical examples are given to show the effectiveness of the given theoretical results.

The rest of this paper is organized as follows. In Section 2, necessary preliminaries and the definition of JLCLFs are presented. Stability analysis for positive switched linear systems is proposed via JLCLFs in Section 3. Two numerical examples are also given to illustrate the main results in Section 4. Finally, Section 5 concludes this paper.

Notations: For any positive integer m, $\langle m\rangle$ is the set of integers $\{1, 2, \dots, m\}$. $\mathbb{R}^n$ is the n-dimensional real vector space with the norm $\|x\| = \max_{i\in\langle n\rangle}\{|x_i|\}$. Say $A \succ 0$ ($\succeq 0, \preceq 0, \prec 0$) if all entries of matrix A are positive (nonnegative, nonpositive, negative). $\mathbb{R}^n_+$ is the set of n-dimensional positive vector. A Metzler matrix is a real square matrix, whose off-diagonal entries are nonnegative.

2. Problem statement and preliminaries

Consider the following continuous-time switched linear system

$$\dot{x}(t) = A_{\sigma(t)}x(t), \quad t \geq 0, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, the piecewise continuous function $\sigma : [0, +\infty) \to \langle m\rangle$ is the switching signal, the $n \times n$-dimensional system matrix $A_k = (a_{ij}^{(k)})$ is Metzler for each $k \in \langle m\rangle$.

We also consider the following switched linear system with time delay

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-\tau), \quad t \geq 0, \tag{2}$$

where x, σ and A_k, $k \in \langle m\rangle$, are defined as above, $\tau > 0$ is a constant delay, $B_k = (b_{ij}^{(k)})$, $k \in \langle m\rangle$, are $n \times n$-dimensional nonnegative matrices.

As usual, system (1) or (2) is said to be positive if, for any nonnegative initial condition and any switching signal, the corresponding state trajectory $x(t) \succeq 0$ holds for any $t \geq 0$. Note that system (1) is positive if and only if A_k is Metzler for each $k \in \langle m\rangle$, and system (2) is positive if and only if A_k is Metzler and B_k is nonnegative for each $k \in \langle m\rangle$ [15].

It is well known that the positive switched system (1) is asymptotically stable under arbitrary switching if it has a CLCLF, i.e., there exists a vector $\xi = (\xi_i) \in \mathbb{R}^n_+$ such that

$$\xi^T A_k \prec 0, \quad k \in \langle m\rangle. \tag{3}$$

However, in many cases condition (3) usually fails to be valid. For example, it may hold $\xi^T A_k \preceq 0$ for $k \in \langle m\rangle$, which implies that the kth subsystem is only stable. In order to study the asymptotic stability of system (1) when each subsystem is only stable, it is necessary to introduce the following weak linear copositive Lyapunov functions (WLCLFs).

Definition 1 (*[25]*). If there exists a vector $\xi = (\xi_i) \in \mathbb{R}^n_+$ such that

$$\xi^T A_k \preceq 0, \quad k \in \langle m\rangle, \tag{4}$$

then $V(t) = \xi^T x(t)$ (or briefly ξ) is called a WLCLF of system (1).

It is obvious that the existence of a WLCLF can only guarantee the stability of system (1). To guarantee the asymptotic stability of system (1), we further introduce the following joint linear copositive Lyapunov functions (JLCLFs).

Definition 2 (*[25]*). If there exists a vector $\xi = (\xi_i) \in \mathbb{R}^n_+$ such that (4) holds, and

$$\xi^T \sum_{k=1}^{m} A_k \prec 0, \tag{5}$$

then $V(t) = \xi^T x(t)$ (or briefly ξ) is called a JLCLF of system (1).

It is noted that a CLCLF must be a JLCLF. The reverse is not true. (for details, please see the numerical example given in Section 3). In [25], we mainly study the existence of JLCLFs for the particular case of $n = 3$. As an application, we use JLCLFs to show the asymptotic stability of system (1) for the particular case of $m = 2$. Following this direction, we will further show that the existence of JLCLFs can still guarantee the asymptotic stability of the general system (1) under appropriate switching. We also extend the main result to the delay system (2).

3. Main results

Set $\lambda_i = \min\{a_{ii}^{(k)} | k \in \langle m\rangle\}$ for $i \in \langle n\rangle$. Based on Definitions 1 and 2, we have the following lemma.

Lemma 1. *If system (1) has a JLCLF, then $\lambda_i < 0$ for $i \in \langle n\rangle$.*

Proof. Assume that the n-dimensional positive vector ξ is a JLCLF of system (1). By (4), we have that $a_{ii}^{(k)} \leq 0$ for $i \in \langle n\rangle$ and $k \in \langle m\rangle$. On the other hand, (4) and (5) imply that there exists at least one index $k_i \in \langle m\rangle$ for any $i \in \langle n\rangle$ such that $a_{ii}^{(k_i)} < 0$. Therefore, $\lambda_i < 0$ for $i \in \langle n\rangle$. $\square$

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