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Observer based minimum variance control of uncertain piecewise affine systems subject to additive noise

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ABSTRACT

In this paper, a class of uncertain piecewise affine (PWA) systems, subject to system and measurement additive noises is studied. The additive noise signals considered here do not vanish at the equilibrium and the uncertainties are norm bounded. The problem of minimizing the bound on the variance of the steady response of uncertain PWA systems, by means of a hybrid observer–controller, is formulated as an optimization problem subject to a number of constraints in the form of matrix inequalities. The derived constraints are obtained by considering a piecewise quadratic Lyapunov function in combination with the general stability conditions regarding the existence of an upper stochastic bound on the steady state variance for a class of stochastic hybrid systems (SHS). Then the uncertain PWA approximation of a practical system with nonlinear dynamics is presented considering system and measurement noises. The uncertainties arise in the form of the difference between the actual nonlinear dynamics and the PWA approximation. Utilizing the introduced methods, a hybrid observer–controller is designed and implemented on the nonlinear system to demonstrate the effectiveness of the proposed controller design procedure.

1. Introduction

Piecewise affine (PWA) systems are a powerful modeling tool for the control of nonlinear systems. These systems constitute a class of hybrid systems and are often encountered in control systems where piecewise-linear components such as dead-zone, saturation, relays and hysteresis are present. PWA systems may also be used to approximate the dynamics of other nonlinear systems. The stability analysis of deterministic PWA systems based on the multiple Lyapunov function method was studied by many authors [1–3]. Also, the state feedback control of PWA systems has attracted many researches [4,5,3] which consider deterministic systems controlled by continuous-time controllers. The simultaneous observer and controller design for deterministic PWA systems was studied by Rodrigues and How [6]. The PWA systems considered here however, are more generalized; as they include both norm bounded uncertainty terms and additive noise terms in both the process and measurement equations.

Many practical systems, including PWA systems, are subject to uncertainties due to modeling error and external disturbances. In the case of using a PWA system to model the behavior of a nonlinear system, the PWA system will be subjected to more approximation error, thus increasing the uncertainty in the model. Several approaches are proposed for the controller design of uncertain PWA systems. Ben Abdallah et al. [7] assumed that the uncertain system parameters belong to a convex polyhedral and utilized the results obtained by Oliveira and Peres [8] and Xu and Xie [9]. It was proved that obtaining a solution that satisfies the related matrix inequality constraints at the vertices of the mentioned convex polyhedral will result

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in obtaining the desired controller gain values and piecewise quadratic Lyapunov function (PQLF) ensuring the closed loop system stability. However, as the number of system states, partitions and uncertainty polyhedral vertices increase, obtaining a solution by this method becomes exponentially difficult. Feng [10] proposed a method in which only the upper bounds of the system uncertainties are included in the inequalities.

Zhang et al. [11] modeled the system uncertainties similar to Feng [10]. The authors further generalized the model by including a one dimensional stochastic noise term that vanishes at the steady state equilibrium point. The piecewise affine controller design problem was then formulated as an optimization problem subject to a number of matrix inequality constraints in order to achieve stochastic stability in the mean square. However this design has limited applicability as in practice the noise terms do not usually vanish at the steady state equilibrium point and, as will be discussed later, achieving stochastic stability in the mean square is impossible.

The inclusion of the additive noise terms in the PWA systems studied here makes them a subset of the more generalized stochastic hybrid systems (SHS). The SHSs comprise a vast range of systems where some randomness is introduced to a hybrid model, with both discrete and continuous states. The randomness in the SHS could be considered in both the evolution of the discrete state and dynamics of the continuous states. Bujorianu and Lygeros [12] proposed a general structure for SHS where the continuous evolution is defined by a stochastic diffusion process and the governing relation for the discrete state is probabilistic. In this framework, both discrete and continuous dynamics could be interconnected, in other words both of them could be dependent on the discrete and continuous states. Given that the design of controller and observer for the general SHS is very challenging in its general format, many methods have been studied that consider a special class of general SHS for stability analysis and controller design.

The Lyapunov based stability theorems for different classes of stochastic hybrid systems have been widely investigated. Dimarogonas and Kyriakopoulos [13] gave stochastic stability theorems for a class of SHS with arbitrary change of discrete state. The proposed theorems are based on conditions that guarantee the stability of the steady state equilibrium point with probability one regardless of the discrete state dynamics and with noise that vanishes at the steady state equilibrium point. It is notable that discarding the discrete dynamics can simplify the conditions but may result in very conservative sufficient conditions.

The asymptotic stability condition for SHS with a Markov process as the discrete evolution dynamics is investigated by Abate et al. [14] where it is assumed that the jumps have probabilistic dwell-time in each discrete state. More recently Xiong et al. [15] derived the stability condition, in the form of Linear Matrix Inequalities (LMI), for switching linear systems. For a class of stochastic hybrid nonlinear systems with Markov jumps, the almost sure exponential stability theorem is given by Yuan and Lygeros [16]. Again there is a condition that requires the noise term to disappear at the steady state equilibrium point. It is remarkable that exponential and asymptotic stability, or stability with probability one, are not achievable in the presence of noise that remains non-zero at the steady state equilibrium point.

Moving on to the controller design for SHS, Dong and Sun [17] have introduced the impulsive output feedback control for a class of nonlinear SHS with Markov jumps. Boukas [18] addressed the static output feedback control for an uncertain singular linear stochastic system with Markov switches, where there are norm bounded uncertainties and the noise signals vanish at the equilibrium. Another output feedback control design via LMI for continuous-time Markovian jump linear systems is proposed by De Farias et al. [19], which again guarantees the exponential stochastic mean square stability.

Most of the literature for SHS stability and control design have not considered state dependent jumps and additive noises that do not vanish in the equilibrium point. Both of the aforementioned properties are required for the stochastic uncertain PWA systems considered here. In the case addressed in this article, asymptotic stability for mean square variance is impossible and only a bound on the second stochastic moment could be obtained. Similarly Rodrigues and Gollu [20] studied the control of a piecewise stochastic system with additive noise, but the common Lyapunov function method is applied and it is assumed that the drift and noise terms are continuous at the boundaries between subsystems and also no upper bound on the variance is given. One of the other close studies in the stability of stochastic hybrid systems with state-dependent switching is given by Wu et al. [21]. The stability theorem approach given is based on the generalization of Barbalat lemma to the stochastic hybrid systems. But still the given stability theorems consider vanishing noise at equilibrium which results in mean square stability.

The state estimation for SHS has also been the topic of many researches. Battilotti [22] proposed a novel observer for SHS with Markov jumps satisfying some dwell-time. This observer guarantees that the state estimation error of the switching dynamics converges asymptotically to zero with probability one. Ahrens and Khalil [23] developed high-gain observers for a class of nonlinear systems with measurement noise. So there is no discrete state in the main system but the observer jumps between two gain values. Le Ny et al. [24] proposed scheduling Kalman filters for the observation of a class of stochastic linear systems where the Kalman gains are obtained via LMI. Most of the studies in observer design, consider Markov jumps in order to simplify the stability analysis. In this article however, state dependent jumps are considered.

To the best of the authors knowledge however, the problem of stabilizing an uncertain stochastic PWA system by means of a piecewise affine observer–controller has never been addressed in the literature. Thus in this article, a systematic method is proposed for simultaneous design of observer and controller for uncertain stochastic PWA systems. In this system, the noises are additive and non-vanishing at the steady state equilibrium point, so the design procedure gives an upper bound on the combined estimation error and controller error variances.

On the other hand, as stated before, the uncertainties in the considered PWA system may arise when the PWA system is utilized to model the behavior of a nonlinear system. If a control law for the nonlinear system is to be designed based