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Finite automata approach to observability of switched Boolean control networks[☆]

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ABSTRACT

In this paper, the observability of switched Boolean control networks (SBCNs) is determined. First, a new concept of weighted pair graphs for SBCNs is defined. Second, the weighted pair graph is used to transform an SBCN into a deterministic finite automaton (DFA). Lastly, the observability of the SBCN is determined by testing the completeness of the DFA. Based on these results, algorithms for determining the observability and the initial state are designed. The computational complexity of this algorithm is doubly exponential in the number of nodes of SBCNs. Furthermore, more effective sufficient or necessary conditions for the observability of SBCNs are obtained directly from weighted pair graphs. The computational complexity of these conditions is exponential in the number of nodes of SBCNs.

1. Introduction

Switched systems consist of a decision layer and a control layer. The former is logical, i.e., discrete, and decides at a given time, which subsystem is activated. The latter usually corresponds to a set of normal control systems. During the recent decades, the study of control-theoretic problems of switched systems has drawn wide attention [1–8], partly because switched systems characterize systems' behavioral pattern very well: first make a decision, then take action.

Normal control systems are usually classified as linear systems and nonlinear systems. It is well known that nonlinear systems possess much more complex dynamics than linear ones. From the viewpoint of dynamical properties, switched linear systems are much more complex than linear systems. For example, controllable subspaces of switched linear systems are characterized in [1]. However, controllable submanifolds outside of controllable subspaces of switched linear systems are found in [7], which shows an essential difference between linear systems and switched linear systems. There are also

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essential differences between switched linear systems and switched nonlinear systems. For example, it is proved in [2,3] that a switched linear system with two subsystems is asymptotically stable iff there is a convex combination of the two subsystems that is also asymptotically stable. However, it is shown that this property does not hold for switched nonlinear systems [6].

The *Boolean network*, introduced first in [9], and then developed by [10,11], etc., is a simple and effective model to describe the behaviors and relationships of cells, protein, DNA and RNA in a biological system, named genetic regulatory networks (GRNs) (cf. [12,13]). Particularly in [12], exogenous perturbation and regulation to biological systems are described as "control", leading to the concept of *Boolean control networks* (BCNs). A BN/BCN is itself simple but reflects the local dynamical interactions of internal nodes (and external nodes). It is pointed out that "One of the major goals of systems biology is to develop a control theory for complex biological systems" [14]. Hence the study of the control problems of BNs/BCNs is of both theoretical and practical importance. Furthermore, in order to estimate real large-scale GRNs more accurately and describe the uncertain factors that affect the updates of GRNs, probabilistic BNs/BCNs (PBNs/PBCNs) emerge [15]. A PBN/PBCN consists of a finite number of BNs/BCNs, and at each time step, the network randomly chooses a sub-BN/BCN to update. If the choice of sub-BNs/BCNs is designable, then the PBN/PBCN can be regarded as a switched BN/BCN.

In this paper, we are concerned with switched Boolean control networks (SBCNs). Why is it interesting to study SBCNs? The control layer of SBCNs is also logic, which is different from switched systems whose control layer is defined on Euclidean spaces or differential manifolds. Actually SBCNs have finitely many states, inputs and outputs. Despite the simplicity, solutions of control-theoretic problems of BCNs may lead to development of new techniques or fusion of different branches of mathematics. For example, the *semi-tensor product* (STP) of matrices [16] is adopted to transform BNs/BCNs to equivalent algebraic forms represented by special Boolean matrices; the Perron–Frobenius theorem is applied to determine the controllability of BCNs [17]; and the theories of finite automata and formal languages are adopted to determine the observability of BCNs [18]. So many developments may also provide new techniques for studying switched systems.

The *controllability* and *observability* of BCNs have been paid wide attention (cf. [14,19–23,18,24–26,17], etc.) during these years. There have been plenty of results on the controllability of BCNs [19,20] and their related extensions [17,23, 27–32]. However, compared to controllability, the problem on the observability of BCNs is more challenging. Due to the nonlinearity of BCNs, four different types of observability have been proposed [19–21,26]. Some of them are dependent of initial states [19,20], some are dependent of inputs [21], while others are not [26]. They are difficult to be unified due to their essential differences. Recently, in [18], we find a connection between the observability of BCNs and the theories of *finite automata* and *formal languages*, and propose a unified approach to design algorithms for determining the four types of observability of BCNs. Later on, the most general observability in [20] is also determined in [33] based on an algebraic method and in [34] based on the STP of matrices. The theories of finite automata and formal languages are among the mathematical foundations of theoretical computer science. Finite automaton theory involves mainly the study of computational problems that can be solved using them. In computational complexity theory, decision problems are typically defined as formal languages, and complexity classes are defined as the sets of the formal languages that can be parsed by machines with limited computational power. For the details, we refer the reader to [35,36].

The main target of this paper is adopting the idea proposed in [18] to deal with the observability of SBCNs, and trying to obtain easily verifiable and more effective methods to determine observability. The observability of SBCNs is first investigated in [37], and only sufficient but not necessary conditions are given. In this paper, we give equivalent conditions. Actually, we use finite automata and formal languages to design an algorithm to determine observability. To this end, we first define a new concept of weighted pair graphs; second use the graph to transform an SBCN to a finite automaton; and lastly use the automaton to determine observability. The computational complexity of this method is doubly exponential in the number of nodes of SBCNs (i.e., exponential in the number of states, inputs and outputs of SBCNs). In our companion paper [38], we use the original idea proposed in [18] to determine the reconstructibility of BCNs in doubly exponential time complexity. Furthermore, directly from the weighted pair graph for reconstructibility, we obtain an exponential time algorithm to determine reconstructibility. The exponential time algorithm is a significant improvement of the doubly exponential time algorithm. So we also try to obtain an exponential time algorithm to determine observability. However, due to the essential difference between the weighted pair graph for reconstructibility and the one for observability (for the details, please see Section 4), an exponential time algorithm to determine observability has not been obtained. In this paper, by adopting the ideas that are different from the ones in [38], we obtain exponential time necessary or sufficient conditions for observability directly from the weighted pair graph. It is proved in [25] that it is **NP**-hard to determine the observability of BCNs, hence it is directly obtained that it is also **NP**-hard to determine the observability of SBCNs. So there exists no polynomial time algorithm for determining the observability of SBCNs unless $\mathbf{P} = \mathbf{NP}$.

The results of this paper are in the framework of an intuitive algebraic form based on the semi-tensor product (STP) of matrices, for such an intuitive algebraic form will help represent weighted pair graphs and finite automata constructed in the sequel. For comprehensive introduction to the STP of matrices, we refer the reader to [39,16]. The remainder of this paper is organized as follows. Section 2 introduces necessary preliminaries about weighted directed graphs, finite automata, formal languages, STP and SBCNs with their algebraic forms. Section 3 studies how to determine the observability and the initial state of SBCNs. Section 4 gives some effective necessary or sufficient conditions for the observability of SBCNs. Section 5 ends up with some concluding remarks.