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Low-complexity quantized switching controllers using approximate bisimulation ☆

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ABSTRACT

In this paper, we consider the problem of synthesizing low-complexity controllers for incrementally stable switched systems. For that purpose, we establish a new approximation result for the computation of symbolic models that are approximately bisimilar to a given switched system. The main advantage over existing results is that it allows us to design naturally quantized switching controllers for safety or reachability specifications; these can be pre-computed offline and therefore the online execution time is reduced. Then, we present a technique to reduce the memory needed to store the control law by borrowing ideas from algebraic decision diagrams for compact function representation and by exploiting the non-determinism of the synthesized controllers. We show the merits of our approach by applying it to a simple model of temperature regulation in a building.

1. Introduction

The use of discrete abstractions or symbolic models has become quite popular for hybrid systems design (see e.g. [1–5]). In particular, several recent works have focused on the use of symbolic models related to the original system by approximate equivalence relationships (approximate bisimulations [6,7]; or approximate alternating simulation or bisimulation relations [8,9]) which give more flexibility in the abstraction process by allowing the observed behaviors of the symbolic model and of the original system to be different provided they remain close. These approximate behavioral relationships have enabled the development of new abstraction-based controller synthesis techniques [10,11].

In this paper, we go one step further by pursuing the goal of synthesizing controllers of lower complexity with shorter execution time and more efficient memory usage for their encoding. For that purpose, we establish a new approximation result for the computation of symbolic models that are approximately bisimilar to a given incrementally stable switched system. This result is the first main contribution of the paper, it differs from the original result presented by [7] mainly by the fact that the expression of the approximate bisimulation relation uses a quantized value of the state of the switched system rather than its full value in [7]. This difference is fundamental for the synthesis of controllers with lower complexity. Indeed, the combination of this new result with synthesis techniques for safety or reachability specifications presented in [11] yields quantized switching controllers that can be entirely pre-computed offline. The online execution time is then greatly reduced in comparison to controllers obtained using the previous existing approximation result. The second main contribution of the paper is to consider the problem of the representation of the control law with the goal of reducing the memory needed for its storage. This is done by using ideas from algebraic decision diagrams (see e.g. [12]) for compact function representation. Also, the non-determinism of the synthesized controllers can be exploited to further simplify the representation of the control law. Finally, we apply our approach to the synthesis of controllers for a simple model of temperature regulation in a

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building. The results on the synthesis of safety controllers appeared in preliminary form in the conference paper [13], those on reachability controllers are new.

2. Symbolic models for switched systems

In this section, we present an approach for the computation of symbolic models (i.e. discrete abstractions) for a class of switched systems. This problem has been already considered by [7]. In the following, we present a slightly different abstraction result that will allow us to synthesize controllers with lower complexity.

2.1. Switched systems

In this paper, we consider a class of switched systems of the form:

$$\Sigma : \ \dot{\mathbf{x}}(t) = f_{\mathbf{p}(t)}(\mathbf{x}(t)), \quad \mathbf{x}(t) \in \mathbb{R}^n, \quad \mathbf{p}(t) \in P$$

where P is a finite set of modes. The switching signals $\mathbf{p} : \mathbb{R}^+ \to P$ are assumed to be piecewise constant functions, continuous from the right and with a finite number of discontinuities on every bounded interval. We use $\mathbf{x}(t, x, \mathbf{p})$ to denote the point reached at time $t \in \mathbb{R}_0^+$ from the initial condition x under the switching signal $\mathbf{p}$. We will assume that the switched system Σ is incrementally globally uniformly asymptotically stable [7]:

Definition 1. The switched system Σ is said to be incrementally globally uniformly asymptotically stable (δ-GUAS) if there exists a KL function[1] β such that for all $t \in \mathbb{R}_0^+$, for all $x, y \in \mathbb{R}^n$, for all switching signals $\mathbf{p} \in \mathcal{P}$, the following condition is satisfied:

$$\|\mathbf{x}(t, x, \mathbf{p}) - \mathbf{x}(t, y, \mathbf{p})\| \le \beta(\|x - y\|, t). \tag{1}$$

Intuitively, a switched system is δ-GUAS if the distance between any two trajectories associated with the same switching signal $\mathbf{p}$, but with different initial states, converges asymptotically to 0. Incremental stability of a switched system can be characterized using Lyapunov functions [7]:

Definition 2. A smooth function $\mathcal{V} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_0^+$ is a common δ-GUAS Lyapunov function for Σ if there exist K_∞ functions $\underline{\alpha}, \overline{\alpha}$ and a real number $\kappa > 0$ such that for all $x, y \in \mathbb{R}^n$, for all $p \in P$:

$$\underline{\alpha}(\|x - y\|) \le \mathcal{V}(x, y) \le \overline{\alpha}(\|x - y\|);$$
$$\frac{\partial \mathcal{V}}{\partial x}(x, y) \cdot f_p(x) + \frac{\partial \mathcal{V}}{\partial y}(x, y) \cdot f_p(y) \le -\kappa \mathcal{V}(x, y).$$

It has been shown in [7] that the existence of a common δ-GUAS Lyapunov function ensures that the switched system Σ is δ-GUAS.

We now introduce the class of labeled transition systems which will serve as a common modeling framework for switched systems and symbolic models.

Definition 3. A *transition system* $T = (X, U, \mathcal{S}, Y, \mathcal{O})$ consists of:

- a set of states X;
- a set of inputs U;
- a (set-valued) transition map $\mathcal{S} : X \times U \to 2^X$;
- a set of outputs Y;
- and an output map $\mathcal{O} : X \to Y$.

T is *metric* if the set of outputs Y is equipped with a metric d. If the set of states X and inputs U are finite or countable, T is said *symbolic* or *discrete*.

An input $u \in U$ belongs to the set of *enabled inputs* at state x, denoted $\text{Enab}(x)$, if $\mathcal{S}(x, u) \ne \emptyset$. If $\text{Enab}(x) \ne \emptyset$, then the state x is said to be *non-blocking*, otherwise it said to be *blocking*. The system is said to be non-blocking if all states are non-blocking. If for all $x \in X$ and for all $u \in \text{Enab}(x)$, $\mathcal{S}(x, u)$ has 1 element then the transition system is said to be *deterministic*.

A *state trajectory* of T is a finite or infinite sequence of states and inputs, $\{(x^i, u^i) |\ i = 0, \dots, N\}$ (we can have $N = +\infty$) where $x^{i+1} \in \mathcal{S}(x^i, u^i)$ for all $i = 0, \dots, N-1$. The associated *output trajectory* is the sequence of outputs $\{y^i |\ i = 0, \dots, N\}$ where $y^i = \mathcal{O}(x^i)$ for all $i = 0, \dots, N$.

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[1] A continuous function $\gamma : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is said to belong to class K_∞ if it is strictly increasing, $\gamma(0) = 0$ and $\gamma(r) \to \infty$ when $r \to \infty$. A continuous function $\beta : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is said to belong to class KL if for all fixed s, the map $r \mapsto \beta(r, s)$ belongs to class K_∞ and for all fixed r, the map $s \mapsto \beta(r, s)$ is strictly decreasing and $\beta(r, s) \to 0$ when $s \to \infty$.