Nonlinear Analysis: Hybrid Systems 12 (2014) 3–19

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

On the fluidization of Petri nets and marking homothecy☆

Estíbaliz Fraca *, Jorge Júlvez, Manuel Silva

Aragón Institute of Engineering Research (I3A), Universidad de Zaragoza, María de Luna 1, 50018, Spain

ARTICLE INFO

Keywords:
Discrete, continuous and hybrid Petri nets
Fluidization
Continuous Petri nets
Homothetic monotonicity
Reachability
Property preservation
Spurious solutions

ABSTRACT

The analysis of Discrete Event Dynamic Systems suffers from the well known *state explosion problem*. A classical technique to overcome it is to relax the behavior by partially removing the integrality constraints and thus to deal with hybrid or continuous systems. In the Petri nets framework, continuous net systems (technically hybrid systems) are the result of removing the integrality constraint in the firing of transitions. This relaxation may highly reduce the complexity of analysis techniques but may not preserve important properties of the original system. This paper deals with the basic operation of fluidization. More precisely, it aims at establishing conditions that a discrete system must satisfy so that a given property is preserved by the continuous relaxation. These conditions will be mainly based on the *marking homothetic behavior* of the system. The focus will be on logical properties as boundedness, B-fairness, deadlock-freeness, liveness and reversibility. Furthermore, testing homothetic monotonicity of some properties in the discrete systems is also studied, as well as techniques to improve the quality of the fluid relaxation by removing spurious solutions.

1. Introduction

Petri nets [1,2], as other formalisms for Discrete Event Dynamic Systems (DEDS), suffer from the *state explosion problem*. Such a problem may render analysis techniques based on exhaustive enumeration computationally infeasible, particularly for large population systems. A promising approach to overcome this difficulty is to relax the original discrete model by explicitly removing the integrality constraint in the firing of transitions. This process is known as *fluidization*, being its result a continuous Petri net (PN) in which both the firing amounts of transitions and the marking of places are non-negative real quantities (see [3,4]).

Continuous PNs allow the use of some polynomial time complexity techniques for several analysis purposes [4]. Unfortunately, continuous nets may not always preserve important properties of the discrete model (first pointed out in [5]). For this reason, it is crucial to study which discrete PN systems can be "successfully" fluidified and which ones not. Moreover, some techniques can be used to improve the fluidization.

At first glance, the simple way in which the basic definitions of discrete models are extended to continuous ones may make us naively think that their behavior will be similar. However, the behavior of the continuous model can be completely different just because the integrality constraint has been dropped. In other words, not all DEDS can be satisfactorily fluidified. Consider, for instance, the net system in Fig. 1(a). If considered as discrete, the system is deadlock-free: from $\boldsymbol{m}_0 = (3, 0)$, both t_2 and t_1 can be fired alternatively, and no deadlock can be reached. However, if considered as continuous, transition t_2 can be fired in an amount of 1.5 from $\boldsymbol{m}_0$, leading to a deadlock marking $\boldsymbol{m}_d = (0, 1.5)$.

☆ This work has been partially supported by CICYT - FEDER project DPI2010-20413, by the grant IT/27 to the GISED research group and the predoctoral grant B174/11 of the Gobierno de Aragón.

* Corresponding author. Tel.: +34 976762472; fax: +34 976761914.
E-mail addresses: efraca@unizar.es (E. Fraca), julvez@unizar.es (J. Júlvez), silva@unizar.es (M. Silva).

1751-570X/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.nahs.2013.11.002

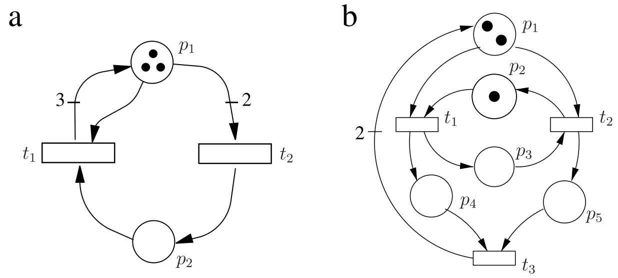

Fig. 1. (a) Not homothetically deadlock-free PN system [5]; (b) homothetic deadlock-free PN system.

Notice that deadlock-freeness of the discrete system in Fig. 1(a) highly depends on its initial marking. In fact, if the initial marking is doubled, i.e., if we consider $\boldsymbol{m}'_0 = (6, 0)$, then the system deadlocks by firing t_2 an amount of 3.

Let us now consider the PN in Fig. 1(b), which exhibits a different behavior. Considered as discrete, it is deadlock-free for $\boldsymbol{m}_0 = (2, 1, 0, 0, 0)$. Moreover, it is deadlock-free for any initial marking proportional to $\boldsymbol{m}_0$, i.e., $\boldsymbol{m}'_0 = k \cdot \boldsymbol{m}_0$, with $k \in \mathbb{N}_{>0}$.

When the PN system is fluidified, i.e., the PN system in Fig. 1(b) is considered as a continuous system, it preserves deadlock-freeness. We will exploit this idea to extract conditions for the preservation of properties.

The present paper explores the kind of features that a discrete net system must exhibit so that a given property is preserved when it is fluidified. It focuses on classical properties as boundedness, B-fairness, deadlock-freeness, liveness and reversibility. The main ideas used here are: (a) the property of homothecy of continuous firing sequences (needed for Lemma 16); (b) the fact that every real number could be approximated by a rational number (used in Lemma 17). Properties preservation is built over these two ideas. Furthermore, homothetic monotonicity of boundedness, B-fairness and deadlock-freeness properties in discrete Petri nets is studied, as well as property preservation for some net system subclasses. Some techniques to improve the fluidization are also considered, where the spurious deadlocks are *removed* with the addition of some *implicit* places.

This work is organized as follows. Section 2 recalls some definitions that will be used in the rest of the paper. Section 3 sets the main results concerning homothetic properties in a discrete net system and its relations with the fluid counterpart. In Section 4, some results about homothetic boundedness and homothetic B-fairness of discrete PN are presented. Section 5 studies whether a discrete PN is homothetically deadlock-free and some techniques for the elimination of spurious deadlocks. Finally, an application example is presented in Section 6, while Section 7 deals with some conclusions.

2. Preliminary concepts and definitions

Some concepts used in the rest of the paper are defined here. In the following, it is assumed that the reader is familiar with discrete Petri nets (see [1,2] for a gentle introduction).

2.1. Petri nets

Definition 1. A PN is a tuple $\mathcal{N} = \langle P, T, \boldsymbol{Pre}, \boldsymbol{Post} \rangle$ where $P = \{p_1, p_2, \ldots, p_n\}$ and $T = \{t_1, t_2, \ldots, t_m\}$ are disjoint and finite sets of places and transitions, and $\boldsymbol{Pre}, \boldsymbol{Post}$ are $|P| \times |T|$ sized, natural valued, incidence matrices.

Given a Petri net and a marking, the discrete Petri net system is defined.

Definition 2. A discrete PN system is a tuple $\langle \mathcal{N}, \boldsymbol{m}_0 \rangle_D$ where $\mathcal{N}$ is the structure and $\boldsymbol{m}_0 \in \mathbb{N}^{|P|}$ is the initial marking.

In discrete PN systems, a transition t is *enabled* at $\boldsymbol{m}$ if for every $p \in {}^\bullet t$, $m[p] \geq Pre[p, t]$. An enabled transition t can be fired in any amount $\alpha \in \mathbb{N}$ such that $0 < \alpha \leq enab(t, \boldsymbol{m})$, where $enab(t, \boldsymbol{m}) = \min_{p \in {}^\bullet t} \lfloor \frac{m[p]}{Pre[p,t]} \rfloor$.

The main difference between discrete and continuous PNs is in the firing amounts and consequently in the marking, which in *discrete* PNs are restricted to be in the naturals, while in *continuous* PNs are relaxed into the non-negative real numbers [3,4]. Thus, a continuous PN system is understood as a relaxation of a discrete one.

Definition 3. A continuous PN system is a tuple $\langle \mathcal{N}, \boldsymbol{m}_0 \rangle_C$ where $\mathcal{N}$ is the structure and $\boldsymbol{m}_0 \in \mathbb{R}^{|P|}_{\geq 0}$ is the initial marking.

In continuous systems, a transition t is *enabled* at $\boldsymbol{m}$ if for every $p \in {}^\bullet t$, $m[p] > 0$. It can be fired in any amount $\alpha \in \mathbb{R}$ such that $0 < \alpha \leq enab(t, \boldsymbol{m})$, where $enab(t, \boldsymbol{m}) = \min_{p \in {}^\bullet t} \{ \frac{m[p]}{Pre[p,t]} \}$.

In both discrete and continuous PN systems, the firing of t in a certain amount α leads to a new marking $\boldsymbol{m}'$, and it is denoted as $\boldsymbol{m} \xrightarrow{\alpha t} \boldsymbol{m}'$. It holds $\boldsymbol{m}' = \boldsymbol{m} + \alpha \cdot \boldsymbol{C}[P, t]$, where $\boldsymbol{C} = \boldsymbol{Post} - \boldsymbol{Pre}$ is the token flow matrix (incidence matrix if $\mathcal{N}$ is self-loop free) and $\boldsymbol{C}[P, t]$ denotes the column t of the matrix $\boldsymbol{C}$. The state (or fundamental) equation, $\boldsymbol{m} = \boldsymbol{m}_0 + \boldsymbol{C} \cdot \boldsymbol{\sigma}$, summarizes the way the marking evolves, where $\boldsymbol{\sigma}$ is the firing count vector (also known as the Parikh vector) associated with the fired sequence σ.