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Hybrid approach for pest control with impulsive releasing of natural enemies and chemical pesticides: A plant–pest–natural enemy model

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ABSTRACT

The agricultural pests can be controlled effectively by simultaneous use (i.e., hybrid approach) of biological and chemical control methods. Also, many insect natural enemies have two major life stages, immature and mature. According to this biological background, in this paper, we propose a three tropic level plant–pest–natural enemy food chain model with stage structure in natural enemy. Moreover, impulsive releasing of natural enemies and harvesting of pests are also considered. We obtain that the system has two types of periodic solutions: plant–pest-extinction and pest-extinction using stroboscopic maps. The local stability for both periodic solutions is studied using the Floquet theory of the impulsive equation and small amplitude perturbation techniques. The sufficient conditions for the global attractivity of a pest-extinction periodic solution are determined by the comparison technique of impulsive differential equations. We analyze that the global attractivity of a pest-extinction periodic solution and permanence of the system are evidenced by a threshold limit of an impulsive period depending on pulse releasing and harvesting amounts. Finally, numerical simulations are given in support of validation of the theoretical findings.

1. Introduction

The effective agricultural pests control in plants is a major concern for the farmers. With the development of society and progress of agriculture science and technology a wide range of pest control methods are available to farmers such as chemical control, biological control, physical control, and remote sensing. Especially for two species predator–prey system or three-species food chain system, two impulsive control methods, biological and chemical controls have been studied by many researchers. Biological control is the suppress in the pest populations from the interaction of other living organisms, often called natural enemies or beneficial species [1–3]. The biological control situation is called successful, if pest population is extinct or maintain at adequately low levels. Specifically for the intention of suppressing the abundance of the pest in a particular target region to a level at which it no longer causes agricultural and economical damages. This can be accomplished by mass reproduction and periodic release of natural enemies of the pest in that region. On the other hand, the chemical control means spreading the pesticide to kill the pest population at every fixed moment of time. Chemical pesticides are useful because they can rapidly destroy a significant portion of pest population and sometimes provide the only feasible method for preventing the economic loss. However, the overuse of chemical pesticides creates many ecological,

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environmental, sociological problems and also recognized as a major health hazard to human being and natural enemies. But there are chemicals which are harmful only for the pest population and harmless for plant and natural enemy population. For example Imidacloprid and Bacillus thuringiensis are safe insecticides to both natural enemy and plant with little Poisson and have a mixed reputation regarding its safety to natural enemies of pests. It has low toxicity to spiders, some predatory beetles, and some predatory bugs. Thus, in this case the biological and chemical control methods can be considered together for the better economical balance.

Further, effective biological control often requires a good understanding of the biology of the pest and life cycle of its natural enemies, as well as the ability to identify various life stages of relevant insects in the plants. In insects, such stages are particularly easy to recognize, being separated by short, clearly identifiable, events such as eggs, larva, moult or pupation. Stage-structured models have already received much attention. The stage-structured population model with immature and mature life stages are studied by many researchers [4–9]. Ma et al. [5] proposed a single-species stage structure population model as follows:

$$\begin{aligned} \dot{N}_i(t) &= B(N(t))N_m(t) - dN_i(t) - \delta(N_i(t)), \\ \dot{N}_m(t) &= \delta(N_i(t)) - dN_m(t), \end{aligned} \tag{1.1}$$

where $N_i(t)$ and $N_m(t)$ denote the immature and mature populations, respectively, δ is the maturity rate, which determines the mean length of the juvenile period, d is the natural death rate of immature and mature populations. Here, B is the birth rate of immature population and $N(t) = N_i(t) + N_m(t)$.

Furthermore, an impulsive differential equation is used as a mathematical tool to study biological and chemical pest control in plant population [10–14] and it is also used to study other type of problems in population dynamics as well as applied science [15–20]. In recent decades, two species stage structured pest–natural system with biological or chemical control is studied by many researchers [21,22,5,4]. Specially, Huang et al. [21] devoted the attention on a stage-structured predator–prey system with impulsive effect and Holling mass defence. In [23], by using theories and methods of ecology and ODE, a two-prey–one-predator system with Watt-type functional response and impulsive perturbations on the predator is established. This system is affected by impulse which can be considered as a control. The system has a rich dynamic behavior including quasi-periodic oscillation, narrow periodic window, wide periodic window, chaotic bands, period doubling bifurcation, symmetry-breaking pitchfork bifurcation, period-halving bifurcation and crises. Moreover, to understand a complex ecological system, it is necessary to study multi-species systems. Many existing biological pest control food chain models, which control pests by impulsively releasing natural enemies or by chemical pesticides, neglect the combined effect of biological and chemical control on pest and also neglect the concept of stage structured population in the food chain model with impulsive perturbation [24–27].

Thus in this paper a stage structured food chain model with simultaneous effect of impulsive biological and chemical control techniques is proposed and investigated. The paper is organized as follows: in Section 2, we propose a plant–pest–natural enemy food chain mathematical model with stage structure and impulsive perturbation. The boundedness and some important lemmas are established in Section 3. In Section 4, local stability and global attractivity of a periodic solution are studied. The threshold of an impulsive period for the permanence of the system is obtained in Section 5. Further, in Section 6, numerical simulations and discussions are given. Finally, conclusions for this paper are given in the last section.

2. Mathematical model

In the modeling process, it is assumed that the plant population $X(\tau)$ grows logistically with the growth rate R_0 and the carrying capacity K_0. The natural enemy has two distinct life stages, e.g. immature and mature. Let $Z_1(\tau), Z_2(\tau)$ be the densities of immature and mature natural enemy populations respectively and $Y(\tau)$ be the density of pest populations in that region of consideration at time τ. Here, it is also assumed that growth of the immature population depends on mature and only mature natural enemy can harvest the pest population. In view of these assumptions and motivated from (1.1), a Lotka–Volterra-type plant–pest–natural enemy food chain model with stage structure is given by the following set of impulsive differential equations:

$$\begin{aligned} \frac{dX(\tau)}{d\tau} &= R_0X(\tau)\left(1 - \frac{X(\tau)}{K_0}\right) - A_1X(\tau)Y(\tau), \\ \frac{dY(\tau)}{d\tau} &= A_1B_1X(\tau)Y(\tau) - A_2Y(\tau)Z_2(\tau) - D_1Y(\tau), \\ \frac{dZ_1(\tau)}{d\tau} &= A_2B_2Y(\tau)Z_2(\tau) - (D_2 + \mu)Z_1(\tau), \\ \frac{dZ_2(\tau)}{d\tau} &= \mu Z_1(\tau) - D_2Z_2(\tau), \end{aligned} \tag{2.1}$$

where A_1 and A_2 are per capita rates of predation; B_1 and B_2 are the product of per capita rate of predation and the conversion rates; D_1 and D_2 are the death rates of pest and natural enemy, respectively. The parameter μ is the maturity rate of natural enemy and all the parameters in the model are positive constants.