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On existence of oscillations in hybrid systems

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The paper extends the notion of oscillations in the sense of Yakubovich to hybrid dynamics. Several sufficient stability and instability conditions for a forward invariant set are presented. The consideration is illustrated by the analysis of a model of two-link compass-gait biped robot.

1. Introduction

Oscillations constitute one of the main operating modes for many systems in nature or in techniques [1–6]. In some cases it is required to maintain the oscillations, in other cases the oscillations have to be suppressed [7,8]. In all cases the conditions of existence of sustained oscillations are of great importance since they allow one to analyze/design a system with desired oscillating properties.

There are many stability theories and definitions of oscillations [8]. Among them in this work we choose one proposed by Prof. Yakubovich almost 40 years ago [9]. This approach is rather generic, and it covers periodical and chaotic oscillations. Contrarily to a pure periodical case, when existence conditions are rather sophisticated [10], the conditions of oscillations in the sense of Yakubovich (Y-oscillations) are simple. For Lurie systems, they are formulated in the frequency domain [9,11,12,4]. For a generic nonlinear system, the conditions of Y-oscillations are given using Lyapunov arguments [13] or applying the homogeneity framework [14]. The main goal of the present paper is to extend the notion of Y-oscillations to hybrid dynamical systems. For this purpose several stability/instability Lyapunov conditions are formulated for hybrid dynamics.

There exist many applications where a system has a hybrid dynamics (continuous and discontinuous) and it is oscillating. The most important one comes from robot locomotion [15,16]. The phenomenon has a hybrid nature due to impacts occurring when a leg is hitting the ground, and the main operating mode is a periodical oscillation. The design/analysis of robot locomotion [15–19] or a sliding-mode system [20] as a periodically oscillating system is rather sophisticated. However, relaxing the periodicity requirement and considering Y-oscillations, it is possible to develop more constructive conditions for the analysis and design of robot motion.

The problem is introduced and illustrated for a two-link compass-gait biped robot model in Section 2. Some preliminaries are given in Section 3. The main result on conditions of Y-oscillations in hybrid systems is presented in Section 4. Several examples of hybrid oscillating systems are given in Section 5.

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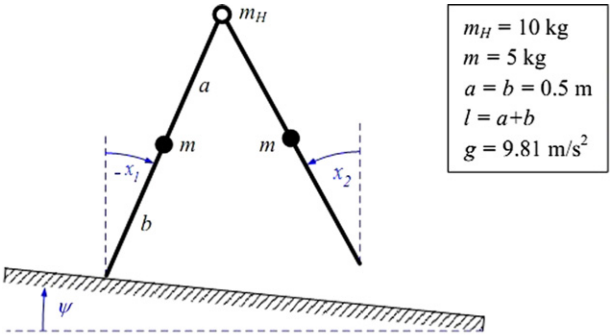

Fig. 1. Schematic of the compass-gait biped on a shallow slope.

2. Motivating example

Dynamics of a two-link compass-gait biped robot with a control torque applied at the hip (schematically shown in Fig. 1) can be described by the hybrid system [21,15]:

$$\begin{cases} \dot{x}_1 = x_2; \\ p_1\dot{x}_2 - p_2(c_{13}\dot{x}_4 + s_{13}x_4^2) - p_4\sin(x_1) = u; \\ \dot{x}_3 = x_4; \\ p_3\dot{x}_4 - p_2(c_{13}\dot{x}_2 - s_{13}x_2^2) + p_5\sin(x_3) = -u, \end{cases} \quad \mathbf{x} \in C;$$
$$\begin{cases} x_1^+ = x_3^-; \\ x_3^+ = x_1^-; \\ A\begin{bmatrix} x_2^+ \\ x_4^+ \end{bmatrix} = B\begin{bmatrix} x_2^- \\ x_4^- \end{bmatrix}, \end{cases} \quad \mathbf{x} \in D, \tag{1}$$

where $\mathbf{x} = [x_1\,x_2\,x_3\,x_4]^T \in \mathbb{R}^4$ is the state vector of the robot, $u \in \mathbb{R}$ is the control torque; $s_{13} = \sin(x_1 - x_3)$, $c_{13} = \cos(x_1 - x_3)$,

$$A = \begin{bmatrix} p_1 - p_2c^- & p_3 - p_2c^- \\ -p_2c^- & p_3 \end{bmatrix}, \qquad B = \begin{bmatrix} p_7c^- - p_6 & -p_6 \\ -p_6 & 0 \end{bmatrix},$$

$c^- = \cos(x_1^- - x_3^-)$; $C = \mathbb{R}^4$, $D = \{\mathbf{x} \in \mathbb{R}^4 : \cos(x_1 + \psi) = \cos(x_3 + \psi)\}$ define the sets with continuous and discrete dynamics respectively; p_i, $i = 1, \ldots, 7$ are the robot parameters related with its physical counterparts presented in Fig. 1 as $p_1 = (m_H + m)l^2 + ma^2$, $p_2 = mlb$, $p_3 = mb^2$, $p_4 = (m_Hl + mb + ml)g$, $p_5 = mbg$, $p_6 = mab$, $p_7 = m_Hl^2 + 2mal$; ψ is the slope of the walking surface. The standard abbreviations

$$x^- = x(t^-) = \lim_{\varepsilon\to 0} x(t - |\varepsilon|), \qquad x^+ = x(t^+) = \lim_{\varepsilon\to 0} x(t + |\varepsilon|)$$

are used to denote the values before and after jumps of a solution. The trajectories of (1) with a unique switch at each passage of D correspond to the physical behavior of a compass-gait biped robot.

For $\psi = 2.87\pi/180$ and $u = 0$ the system has two (known) nontrivial periodic solutions [15]. If the value of ψ is changed or if it is a time-varying sequence (that corresponds to a walking on an irregular surface), then the conditions of existence of periodical solutions is an open question. That is more, it is hard to address this problem with the existing theoretical approaches for a time-varying surface slope.

Tools for the detection and stabilization of periodical oscillating modes for a constant ψ are largely reported in the literature (see [21,15–18] and references therein). These analysis and design methods are complex and, frequently, only local. The problem complexity is originated by the hybrid nature of the dynamics. It is hard to search, analyze and stabilize such a type of behavior. However, if we would skip the requirement that the motion has to be periodical, allowing other types of "oscillating behavior" (that is a natural relaxation for a time-varying ψ), then existence conditions could be more simple and global depending of course on an alternatively defined oscillation concept. Below such a reduction is demonstrated applying Y-oscillation concept.

3. Preliminary results

This section has three parts. The first one deals with the hybrid system formalism following [22–24]. The second part introduces (pre)asymptotic stability definition and its equivalent Lyapunov characterization from [23]. The third part is devoted to sufficient conditions of Lyapunov-type instability for hybrid systems.

In this work, $\mathbb{R}$ denotes the real numbers, $\mathbb{R}_+$ corresponds to the nonnegative real numbers, $\mathbb{Z}$ and $\mathbb{Z}_+$ are stated for integers and nonnegative integers respectively. The symbol $|\cdot|$ denotes an absolute value for a real scalar or Euclidean norm