



# Robust fault detection for a class of uncertain switched nonlinear systems via the state updating approach



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## ABSTRACT

In this paper, the fault detection problem for a class of state-dependent switched nonlinear systems with linear switching surface is addressed. The investigation of fault detection problem includes two parts: design sub-filters for each subsystem, and determine a proper update of estimated state. A fault detection filter is proposed incorporating the update of estimated state at switching instants and the multiple Lyapunov function approach is employed in the design process to reduce the conservativeness. It should be pointed out that the state update relation is derived based upon multiple Lyapunov functions and also on the information of switching surface. In the end, a special case in which the state space is divided into several polyhedral cells is discussed. A numerical example is given to illustrate the effectiveness of proposed results.

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## 1. Introduction

Switched systems are dynamical systems composed of a family of subsystems described by differential of difference equations with a switching rule orchestrating the switching between the subsystems. As a typical subclass of hybrid systems, it has attracted much attention of researchers and engineers in the field of control systems during recent decades. Switched systems can be efficiently used to model many practical systems which are inherently multi-model in the sense that multiple dynamics describe their behavior. Many physical processes exhibit switched and hybrid nature [1–3], and switched systems arise in many engineering applications [4–6]. Furthermore, more and more engineering applications resort to switching strategy to improve control performance [1,7,8]. Stability and stabilization are the main concerns in the field of switched systems and many variations of Lyapunov function are effectively utilized to address this issue [9–12]. Dwell time and average dwell time approaches have been employed for the case of time-dependent switched systems [13–15]. To have a review of stability and stabilization issues related to switched and hybrid systems, readers are referred to [16,17], and the references cited therein.

On the other hand, model-based fault detection has been widely applied in aerospace, chemical and automotive industries [18–20]. The basic idea of fault detection is to construct a residual signal and, based on this, to determine a residual evaluation function to compare with a predefined threshold. When the residual evaluation function has a value larger than the threshold, an alarm is generated. The fault detection problem is hot topic of researchers from many decades and many significant results have been developed, e.g. [21–25], etc. Recent results concerned with signal filtering, fault detection, diagnosis and fault tolerant control for switched systems can be found in some research articles [26–38] and surveys [39,40]. However, most of these results focus on arbitrary, time-dependent and stochastic switched systems. Another important

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class labeled as ‘state-dependent switched system’ (switched system with state-dependent switching law), in particular the system with switching surface, also requires special attention when it comes to modeling and simulation with proper treatment of discrete events [41,42]. However, to the best of our knowledge, few results about fault detection for state-dependent switching system have been reported. This motivates the present investigation.

In this paper, a special type of state-dependent switched system is considered where the switching from one mode to another is triggered by the switching surface defined as linear hyper planes. If the number of subsystems is limited as is the case for most practical application, it is reasonable to design a switched filter by designing a collection of filters for each subsystem. If the information about the active subsystem in running time is known, the corresponding sub-filter can be synchronously activated. This type of switched fault detection filter can easily be obtained by the standard common Lyapunov function approach. However it usually exhibits overly conservativeness in the design procedure. As a well-known successful replacement to common Lyapunov approach, the multiple Lyapunov function approach is developed. To apply the less conservative multiple Lyapunov function idea into fault detection filter design, a novel type of fault detection filter is proposed by monitoring a proper filter state update behavior. Here it is notable that, by comparing with reported results about time-dependent switched systems [43,44] and stochastic switched systems [45–47], the main novelty for state-dependent switched system lies in the update relation of filter state. Key points in proposed fault detection filter design are: (1) a proper update of filter state at each switching instant is one of key ingredients in filter design. In our paper, the proper update of estimated state is based on the information of the switching surface and current value of estimated state and derived by a constructive method. (2) Using the multiple Lyapunov function approach is the second crucial point in our filter design. With the aid of the estimated state update, the fault detection filter design is formulated as the LMIs-based problem which is convenient for the design procedure with the aid of existing efficient numerical software tools.

Keeping in mind the above two points, the fault detection filter designed through the multiple Lyapunov function approach provides us an improved and less conservative result. Moreover, the case in which the state space is divided into several polyhedral cells is particularly discussed as a special case.

The rest of this paper is organized as follows. In Section 2, problem formulation and some preliminaries are introduced. The main result, fault detection for state-dependent switched nonlinear systems with linear switching surface, is given in Section 3. Further discussion on a special case is considered in Section 4. A numerical example is given in Section 5 and conclusions are given in Section 6.

*Notations:* the notations used in this paper are fairly standard. The superscript “ $T$ ” stands for matrix transposition,  $\mathbb{R}^n$  denotes the  $n$  dimensional Euclidean space and  $\mathbb{R}^+$  represents the set of nonnegative real numbers, the notation  $\|\cdot\|$  refers to the Euclidean norm and  $\|\cdot\|_2$  is the  $\mathcal{L}_2$ -norm. In addition, in symmetric block matrices, \* is used to indicate the terms that are introduced by symmetry and  $\text{diag}\{\cdot\cdot\cdot\}$  stands for a block-diagonal matrix. The notation  $P > 0$  ( $P \geq 0$ ) means  $P$  is real symmetric and positive definite (semi-positive definite).  $I$  stands for the identity matrix.

## 2. Problem formulation and preliminaries

Consider a class of continuous-time uncertain switched nonlinear systems  $\Sigma$ , described by following differential equations

$$\dot{x}(t) = A_i(t)x(t) + \Psi_i(x, u) + B_i^d(t)d(t) + B_i^f(t)f(t) \tag{1a}$$

$$y(t) = C_i(t)x(t) + D_i^d(t)d(t) + D_i^f(t)f(t) \tag{1b}$$

where  $x(t) \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}^m$  is the input vector,  $y(t) \in \mathbb{R}^q$  is the measurement vector of the system,  $d(t) \in \mathbb{R}^p$  is disturbance,  $f(t) \in \mathbb{R}^l$  is the fault to be detected.  $d(t)$  and  $f(t)$  are assumed to be  $\mathcal{L}_2$ -norm bounded. Matrices  $A_i(t) = A_i + \Delta A_i(t)$ ,  $B_i^d(t) = B_i^d + \Delta B_i^d(t)$ ,  $B_i^f(t) = B_i^f + \Delta B_i^f(t)$ ,  $C_i(t) = C_i + \Delta C_i(t)$ ,  $D_i^d(t) = D_i^d + \Delta D_i^d(t)$ ,  $D_i^f(t) = D_i^f + \Delta D_i^f(t)$  are time-varying, in which  $A_i, B_i^d, B_i^f, C_i, D_i^d, D_i^f$  are constant matrices with appropriate dimensions and  $\Delta A_i(t), \Delta B_i^d(t), \Delta B_i^f(t), \Delta C_i(t), \Delta D_i^d(t), \Delta D_i^f(t)$  are unknown matrices representing system parameter uncertainties and are assumed to be of the form

$$\begin{bmatrix} \Delta A_i(t) & \Delta B_i^d(t) & \Delta B_i^f(t) \\ \Delta C_i(t) & \Delta D_i^d(t) & \Delta D_i^f(t) \end{bmatrix} = \begin{bmatrix} M_{i1} \\ M_{i2} \end{bmatrix} F(t) \begin{bmatrix} N_{i1} & N_{i2} & N_{i3} \end{bmatrix} \tag{2}$$

where  $M_{i1}, M_{i2}, N_{i1}, N_{i2}, N_{i3}, N_{i4}$  are known real constant matrices and  $F(t)$  is an unknown real-valued time varying matrix satisfying  $F^T(t)F(t) \leq I, \forall t \in \mathbb{R}^+$ . It is worth noting that the parameter uncertainty structure described as above has been widely used in the problems of robust control and robust filtering for both continuous and discrete systems and can capture parameter uncertainty in several practical situations [48].

For all  $i \in \mathcal{I}$ , nonlinearities  $\Psi_i(x, u)$  are assumed to be known nonlinear functions satisfying initial state condition  $\|\Psi_i(0, u^*)\| = 0$  and the following global Lipschitz condition with respect to  $x$

$$\|\Psi_i(x_1, u^*) - \Psi_i(x_2, u^*)\| \leq \alpha_i \|x_1 - x_2\| \quad \forall x_1, x_2 \in \mathbb{R}^n \tag{3}$$

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