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Stochastic stability and stabilization of positive systems with Markovian jump parameters

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ABSTRACT

This paper is concerned with stochastic stability and stabilization of positive systems with Markovian jump parameters in both continuous-time and discrete-time contexts. First, stochastic stability of the underlying systems in the autonomous case is discussed. Then, stochastic stabilization of non-autonomous systems is addressed, and mode-dependent state-feedback controllers are designed. All the proposed conditions are solvable in terms of linear programming with additional parameters. Finally, numerical examples are given to show the effectiveness of the present design.

1. Introduction

In the control community, there exists a class of systems termed as positive systems. This class of systems has found many applications in practice, such as economics, biology, chemistry, sociology, communications etc. The relevant theory on positive systems can refer to [1–8] and so on. Quite recently, new studies in formation flying [9], communication systems [10,11], medical [12], and other areas, have shown the importance of positive switched systems. Consequently, the studies on positive switched systems have become a hot issue due to their fundamental importance from both theoretical and practical viewpoints. Stability and stabilization are two basic problems which have been paid much attention [13–15], and it has been shown that the so-called linear copositive Lyapunov function is more effective for solving corresponding control problems of positive systems than traditional quadratic Lyapunov functions. Accordingly, the linear programming technique is more efficient than the classic linear matrix inequality approach.

It is well known that many dynamic systems including positive switched systems are often subject to abrupt changes, for instance, component failures, sudden environment changes, or some other unexpected factors. Markovian jump systems provide a choice to model the abrupt phenomena. Many researchers have devoted themselves to the research on Markovian jump systems, and a number of valuable results have been obtained [16–24], to list a few. Positive systems with Markovian jump parameters are a special class of Markovian jump systems. Such a class of systems provide a unified framework for mathematical modeling of many dynamic systems. Network employing TCP in communication systems can be described by positive systems with a hidden Markovian process [11,25,26]. In competitive electricity markets, there exist many uncertainty variations such as energy demand, availability of generation units and fuel costs, as well as participants are faced with the uncertainty of their competitors' behavior. The analysis of electricity price time series reflects a switching nature, which can be represented by a set of dynamic models sequenced together by a Markov chain [27]. These examples illustrate

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that theoretical findings of positive systems with Markovian jump parameters would have great potential applications in practical systems.

By the above observations, we find that positive systems with Markovian jump parameters have extensive applications in practice. However, so far, there are few results [28] concerning positive systems with Markovian jump parameters and existing results mainly focus on positive switched systems [14,29–32]. It should be noted that the stabilization problem of positive switched systems is theoretically challenging all the time, since one needs to consider the stability of not only subsystems and the close-loop systems under switching effects but also the positivity of subsystems. Furthermore, it should be also noted that many well-established results and methods for the stabilization of general switched systems cannot be directly applied to multi-mode positive systems due to the positivity. In [28], stability and stabilization of positive systems with Markovian switching were investigated by using linear matrix inequalities. As described earlier, the linear matrix inequalities based strategy may not be effective than the linear programming strategy. Therefore, it is of theoretical and practical significance to consider stochastic stability and stabilization problems of positive systems with Markovian jump parameters. These motivate us to carry out the present work.

In this paper, we focus on stochastic stability and stabilization of positive systems with Markovian jump parameters. First, sufficient conditions of stability for such systems are derived, upon which, the stabilization problem is investigated by using the stochastic linear copositive Lyapunov function approach. Then, state-feedback controllers are designed to stabilize the underlying systems. All the conditions are formulated in linear programming terms with additional parameters. The rest of the paper is organized as follows. Section 2 gives some preliminaries. In Section 3, the stability problem is studied. The stabilization is solved in Section 4. Numerical examples are provided in Section 5. Section 6 concludes the paper.

Notation: $\mathbb{N}$ and $\mathbb{N}_+$ are the sets of nonnegative and positive integers, respectively. $\Re$, $\Re^n$, and $\Re^{n\times n}$ are the sets of real numbers, the vector of n-tuples of real numbers, and the space of $n\times n$ matrices with real entries, respectively. $\|\cdot\|$ and $\|\cdot\|_2$ represent the 1-norm and the Euclidean norm, respectively, and are denoted by $\|x\| = \sum_{i=1}^{n} |x_i|$ and $\|x\|_2 = (\sum_{i=1}^{n} x_i^2)^{1/2}$, where $x = (x_1, \dots, x_n)^T$. A^T is the transpose of A. I_n is the $n \times n$ identity matrix. For v in $\Re^n$, v_i is the ith component of v, and $v \succ 0$ ($\succeq 0$) means $v_i > 0$ (≥ 0) for $i = 1, 2, \dots, n$. $\overline{\mu}_v$ stands for the maximal element of v. Similarly, $\underline{\mu}_v$ stands for the minimal element of v. For a matrix A in $\Re^{n\times n}$, a_{ij} stands for the element in the ith row jth column of A, and $A \succ 0$ ($\succeq 0$) implies $a_{ij} > 0$ (≥ 0) for $i = 1, 2, \dots, n$, $j = 1, 2, \dots, n$. A matrix A is said to be a Metzler matrix if its off-diagonal elements are all nonnegative real numbers.

2. Problem formulation and preliminaries

Consider the continuous-time and discrete-time positive systems described by

$$\dot{x}(t) = A(r_t)x(t) + B(r_t)u(t), \tag{1}$$

and

$$x(k+1) = A(r_k)x(k) + B(r_k)u(k), \tag{2}$$

respectively, where $x(t) \in \Re^n$ and $u(t) \in \Re^m$ (or $x(k) \in \Re^n$, $u(k) \in \Re^m$) are the system state and control input, respectively. The $\{r_t, t \geq 0\}$ (or $\{r_k, k \geq 0\}$) represents the jumping process and takes values in a finite set $S = \{1, 2, \dots, N\}$, $N \in \mathbb{N}_+$. For continuous-time systems, the jumping process is a continuous-time, discrete-state homogeneous Markov process and has the transition probabilities

$$P(r_{t+\Delta} = j | r_t = i) = \begin{cases} \lambda_{ij}\Delta + o(\Delta), & \text{if } j \neq i \\ 1 + \lambda_{ii}\Delta + o(\Delta), & \text{if } j = i \end{cases}$$

where $\Delta > 0$, $\lim_{\Delta\to 0}(o(\Delta)/\Delta) = 0$, $\lambda_{ij} \geq 0$ $(i, j \in S, j \neq i)$, and $\lambda_{ii} = -\sum_{j=1, j\neq i}^{N} \lambda_{ij}$. For discrete-time systems, the jumping process is a discrete-time homogeneous Markov chain with the transition probabilities

$$P(r_{k+1} = j | r_k = i) = \pi_{ij}$$

where $\pi_{ij} \geq 0$, $i, j \in S$, and $\sum_{j=1}^{N} \pi_{ij} = 1$. For $r_t = i \in S$ (respectively, $r_k = i \in S$), the system matrices of the ith mode are denoted by (A_i, B_i), where $A_i \in \Re^{n\times n}$ and $B_i \in \Re^{n\times m}$ for $i \in S$. Throughout the paper, it is assumed that, for system (1), A_i is a Metzler matrix and $B_i \succeq 0$ for each $r_t = i \in S$, and for system (2), $A_i \succeq 0$ and $B_i \succeq 0$ for each $r_k = i \in S$.

Next, some definitions and lemmas are recalled for later development.

Definition 1. Consider a continuous-time system

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{3}$$

and a discrete-time system

$$x(k+1) = Ax(k) + Bu(k). \tag{4}$$

System (3) is said to be positive if for any initial condition $x(t_0) \succeq 0$ and $u(t) \succeq 0$ all states $x(t) \succeq 0$ for all nonnegative time; System (4) is said to be positive if for any initial condition $x(k_0) \succeq 0$ and $u(k) \succeq 0$ all states $x(k) \succeq 0$ for all $k \geq 0$.