Nonlinear Analysis: Hybrid Systems 14 (2014) 74–85

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

$\mathcal{H}_\infty$ filtering for a class of discrete-time switched fuzzy systems

Lixian Zhang [a,b,*], Xianke Dong [a], Jianbin Qiu [a], Ahmed Alsaedi [c], Tasawar Hayat [c,d]

[a] Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150001, China

[b] Communication Systems and Networks (CSN) Research Group, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

[c] Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

[d] Department of Mathematics, Quaid-i-Azam University, Islamabad 44000, Pakistan

ARTICLE INFO

Article history:
Received 23 December 2012
Accepted 23 May 2014

Keywords:
Mode dependent average dwell time
$\mathcal{H}_\infty$ filtering
Switched systems
Fuzzy systems

ABSTRACT

This paper focuses on $\mathcal{H}_\infty$ filtering problem for a class of discrete-time switched nonlinear system under both fast and slow switching property. Each nonlinear mode of the switched system is expressed by a set of linear systems in local regions via T–S fuzzy modeling. Based on mode-dependent and fuzzy-basis-dependent Lyapunov functions, the existence conditions for the desired full-order filters are derived such that the developed filtering error system is globally uniformly asymptotically stable with a given $\mathcal{H}_\infty$ performance index. In particular, the mode-dependent average dwell time switching scheme is proposed for slow switching to relax the restrictions of average dwell time. Finally, the validity and potential of the developed theoretical results are demonstrated by a numerical example.

1. Introduction

Fuzzy model-based control provides a systematic approach to tackle nonlinear control systems. Based on T–S fuzzy model, smooth nonlinear systems can be approximated by a blending of local linear system models, and some sophisticated theories in the linear field can be fully transplanted into nonlinear systems [1–4]. During the past decades, T–S fuzzy systems have been extensively investigated, and many control and filtering issues related to T–S fuzzy systems have been studied in the literature, see, e.g., [5–11] and the references therein. For many of the earlier researches, the common Lyapunov function was widely employed because of its effectiveness and convenience in stability analysis and synthesis of control systems, but such common Lyapunov matrix does not exist for many highly nonlinear complex systems. This motivates the development of the more advanced fuzzy basis-dependent Lyapunov functions-based approach which is less conservative [12,13].

On the other hand, switched systems can be used to describe many practical engineering systems. Since numerous practical or man-made systems display switching features, switched systems have a wide range of applications [14–19]. Switching signals, the crucial factor in analysis and design of switched systems, are often classified into autonomous and controlled ones. As a typical controlled switching rule, ADT switching scheme demands that the average time interval between any two consecutive switching instances is no less than τ_a [20–24], which is determined by two mode-independent parameters, the increase coefficient and decay rate of the Lyapunov-like function. However, such two common parameters may give rise to

* Corresponding author at: Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150001, China. Tel.: +86 402350 2000.

E-mail address: lixianzhang@hit.edu.cn (L. Zhang).

http://dx.doi.org/10.1016/j.nahs.2014.05.009
1751-570X/

conservativeness to some degree. Considering the limitation, in [25], the concept of average dwell time (ADT) is extended to the concept of mode-dependent average dwell time (MDADT) in order to release this restrictions.

Succeeding the remarkable developments in theory and applications, recently switched systems have been extended further to encompass switched fuzzy systems[26–29]. Switched fuzzy system is a class of switched nonlinear systems, each mode of which is expressed by a set of T–S fuzzy subsystems. The structure of this system can be legibly classified into two levels. The switched upper level which extracts modes from the intricate switched nonlinear system characterizes randomicity, while the fuzzy lower level which divides each mode into several linear subsystems represents nonlinearity. Thus, switched fuzzy system has a huge potential for more precise representation of the dynamics of intricate systems as well as their interactions and couplings.

Therefore, during the last several years, switched fuzzy system has attracted considerable attention. Based on the common Lyapunov function technique, the problems of stability analysis and control synthesis were investigated in [26], but the results were derived from state-dependent approaches which may lead to chattering between modes. More relax stabilization conditions for discrete-time switched fuzzy systems by means of fuzzy-basis-dependent Lyapunov function have been presented in [29]. In addition, [27] investigated the stabilization conditions to design the switched fuzzy controller of switched nonlinear systems based on mode-dependent Lyapunov function. The $\mathcal{H}_\infty$ filter design for continuous switched fuzzy systems with partially unknown membership functions is addressed in [30]. [31] employed the differential Petri net to represent both the discrete switching logic and fuzzy dynamic process. However, to our best knowledge, most of the researches in this area are concerned with system modeling, stability analysis and controller design. The filtering problem of switched fuzzy systems has not been fully investigated, which motivates us to conduct this study.

This paper investigates the problem of $\mathcal{H}_\infty$ filtering for a class of discrete-time switched fuzzy systems under both arbitrary switching signal and slow switching signal. The newly proposed MDADT switching logic is adopted to reduce the conservatives. Moreover, by employing the fuzzy-basis-dependent and mode-dependent Lyapunov function, more relaxed criteria which guarantee the developed filter error system globally uniformly asymptotically stable (GUAS) with a given $\mathcal{H}_\infty$ performance are obtained. The remainder of the paper is organized as follows. The problems of the study are formulated and some necessary preliminaries are recalled in Section 2. In Section 3, the l_2-gain analysis and $\mathcal{H}_\infty$ filter design for discrete-time switched nonlinear system under arbitrary switching signal is developed. Then the corresponding results are extended to the MDADT switching scheme. Section 4 provides an illustration example and Section 5 concludes the paper.

Notation: the notation in this paper is fairly standard. The subscript "T" and "−1" stand for matrix transposition and inverse respectively. $\mathcal{R}^n$ denotes the n dimensional Euclidean space. $\|\cdot\|$ refers to the Euclidean vector norm. In addition, in symmetric block matrix or long matrix expressions, we use "$*$" as an ellipsis for the terms that are introduced by symmetry, $diag\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. $l_2[0,\infty)$ is the space of square summable infinite sequence. $\mathcal{C}^1$denotes the space of continuously differentiable functions, and a function $\beta : [0,\infty) \to [0,\infty)$ is said to be of class κ_∞ if it is continuous, strictly increasing, unbounded, and $\beta(0) = 0$. The notation $P > 0\,(P \geq 0)$ means that P is symmetric and positive (semi-positive) definite. I and 0 represent, respectively, identity matrix and zero matrix.

2. Problem formulation and preliminaries

This section discusses the modeling of discrete-time switched fuzzy system, and introduces some necessary definitions and lemmas on switched systems. A general class of nonlinear systems can be expressed by a blending of a set of local linear models via the T–S fuzzy method. Thus, we consider using the following T–S fuzzy models to represent a class of discrete-time switched nonlinear systems.

IF ξ_{1k} is M_{ip1} and $\cdots \xi_{rk}$ is M_{ipr}, THEN

$$\begin{cases} x_{k+1} = A_{ip}x_k + B_{ip}\omega_k \\ y_k = C_{ip}x_k + D_{ip}\omega_k \\ z_k = H_{ip}x_k + L_{ip}\omega_k \end{cases} \tag{1}$$

where $x_k \in \mathcal{R}^n$ is the state vector, $\omega_k \in \mathcal{R}^m$ is the disturbance input which belongs to $l_2[0,\infty)$, $y_k \in \mathcal{R}^l$ is the measurement output, $z_k \in \mathcal{R}^v$ is the signal to be estimated. i is mode determined by switching signal σ which takes its values in $\mathcal{I} = \{1,\ldots,N\}$, $N > 1$ is the number of mode. The appropriately dimensioned matrices $A_{ip}, B_{ip}, C_{ip},\ D_{ip}, H_{ip}$ and L_{ip} denote the pth local model of the ith subsystem. $M_{ipj}, j = 1,\ldots,r$ are fuzzy sets and $\xi_{jk}, j = 1,\ldots,r$ are premise variables.

Then the global mode of ith subsystem is inferred as follows,

$$\begin{cases} x_{k+1} = \sum_{p=1}^{\beta(i)} h_{ip}[A_{ip}x_k + B_{ip}\omega_k] \\ y_{k+1} = \sum_{p=1}^{\beta(i)} h_{ip}[C_{ip}x_k + D_{ip}\omega_k] \\ z_k = \sum_{p=1}^{\beta(i)} h_{ip}[H_{ip}x_k + L_{ip}\omega_k] \end{cases} \tag{2}$$