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Global stabilization of switched control systems with time delay

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ABSTRACT

In this paper, the stabilization problem of switched control systems with time delay is investigated for both linear and nonlinear cases. First, a new global stabilizability concept with respect to state feedback and switching law is given. Then, based on multiple Lyapunov functions and delay inequalities, the state feedback controller and the switching law are devised to make sure that the resulting closed-loop switched control systems with time delay are globally asymptotically stable and exponentially stable.

1. Introduction

A switched system is a hybrid system comprised of continuous-time or discrete-time subsystems and a rule that supervises the switching between subsystems. Switched systems can be found in many areas, such as computer science, control systems, electrical engineering and technology, automotive industry, and air traffic management and control [1–4]. For switched systems, most important and challenging problems are on stability and stabilization (i.e., is it possible to design (or find) a switching law under which the resulting switched systems are stable?). Hence, the recent focus of switched control systems is on the design of a switched law and a controller under which the controlled systems are stable. In the last two decades, stability analysis of switched systems and switching control design have attracted considerable attention among control theorists, computer scientists, mathematicians and practicing engineers. Many interesting and important results have been established. See, for example, [3–14] and the references therein.

On the other hand, time delay phenomenon exists in many practical switched systems, see, for example, [9,10,14,15]. Stability and stabilization problem is also an important and challenging problem for switched systems with time delay. In [15], exponential stability of switched systems with time-delay is established based on average dwell time and Lyapunov functions methods. Using a multiple Lyapunov function method, exponential stability of some special linear switched system is investigated in [16]. Moreover, a linear matrix inequality (LMI) method is applied to study the stability problem of switched systems in [17–20]. In [21], asymptotic stability and stabilization of a class of switched control systems is studied, where a delay-dependent stability criterion is formulated in term of LMIs by using quadratic Lyapunov functions and inequality analysis technique. For discrete-time systems, some interesting results can be found in [4,22,23]. Other methods, such as dwell time and average dwell time, are used in the study of switched systems. For example, stability of some slow-switched control systems has been studied in [24], and stabilization problems for switched systems have been discussed in [25–30].

For the results mentioned above, the switching signal does not involve time delay. However, in real world, a switched control system may have several controllers and not all the controllers are required to switch at the same time. This class of

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systems can be described by switched control systems in which the switching signal has time delay. These switched control systems are much more complex than a conventional switched system and hence only few works are available in the literature, such as, [31,32], where some sufficient conditions for stability are derived for some switched linear systems with time delay appearing in the switching signal. It appears that no results on the stabilization problem are available in the literature for nonlinear switched control systems with distributed time delays and time delay appearing in the switching signal. This has motivated our research. In addition, our results obtained can be applied to the cases with asynchronous switching in actual operation, such as the stabilization of chemical systems [33] and multi-agent systems [34–36]. The asynchronous switching in systems is always caused by the controller lags of the system.

In this paper, we consider static state feedback control for nonlinear switched system with distributed time delays and with time delay appearing in the switching signal. In this study, the time delay of the state may be different from the time delay appearing in the switching signal of the feedback controller. We first introduce some new concepts on stabilization in relation to controller and switching law. Then, by using the method of Lyapunov functions and delay inequalities, some delay-dependent conditions are derived for the design of state feedback controller and a switching law to guarantee the stability of the resulting closed-loop switched control systems. The advantages of this paper are twofold. First, the system model includes both integral terms, called distributed delay, and asynchronous control time lags. This model can cover most of the existing models for switched linear/nonlinear systems. Second, the stability issue under investigation is with respect to both state feedback control and switching signal, which is a much more general problem than previous results.

The outline of the paper is as follows. Section 2 presents some definitions and some technical lemmas needed for the proof of the main results. The design of a switching law for global asymptotical or exponential stabilizability of linear and nonlinear switched control systems are obtained in Sections 3 and 4, respectively. Finally, some concluding remarks are made in Section 5.

2. Preliminaries

The following switched control system is considered:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u + f_{\sigma(t)}(x(t), x_t), \tag{1}$$

where $x(t) \in R^n$, $u \in R^n$ is state feedback control, $A_{\sigma(t)} \in R^{n\times n}$, and $B_{\sigma(t)} \in R^{n\times n}$ are state matrices, the right continuous function $\sigma(t) : R \to \Theta = \{1, 2, \ldots, N\}$ is the switching signal, $x_t \in C_\tau = \{\varphi|\varphi \in C([-\tau, 0], R^n)\}$, $\tau > 0$ is a constant and $f_{\sigma(t)} \in C(R^n \times C_\tau, R^n)$ satisfy $f_{\sigma(t)}(0, 0) = 0$.

In this paper, $N_+ = \{1, 2, \ldots\}$, $R_+ = [0, +\infty)$, E is an identity matrix of appropriate dimension, $|A|$ denotes the usual norm of a matrix $A \in R^{n\times m}$, $\|x_t\| = \sup_{t-\tau\le\theta\le t} |x(\theta)|$ denotes the sup norm of the function $x_t \in C_\tau$. If $A \in R^{n\times n}$, $\lambda(A)$ denotes eigenvalue of A. Let $\sigma = \{(i_1, t_1), \ldots, (i_k, t_k), \ldots\}$ be the switching law, meaning that when $t \in [t_k, t_{k+1})$, the i_{k+1}th subsystem is active, i.e.

$$\dot{x}(t) = A_{k+1}x(t) + B_{k+1}u + f_{k+1}(x(t), x_t), \quad t \in [t_k, t_{k+1}).$$

Let $x(t, t_0, \varphi, \sigma)$ be the solution of system (1) under the switching law σ, starting from (t_0, φ).

It is well known that, a switching law often plays an important role in the study of the stability of a switched system. Even if the subsystems are stable, different switching laws may result in totally different properties of the overall switched systems. In this paper, our focus is on to design a controller u and a switching law σ so that the resulting closed-loop switched systems are guaranteed to possess desired properties.

Next, we propose some concepts on the stabilization for the switched control system.

Definition 2.1. Consider the switched control system (1).

(1) It is said to be stabilizable with respect to (w.r.t.) the state feedback control u and the switching law σ (SWUS) if it is stable under the state feedback u and the switching law σ. That is, for any $\varepsilon > 0$, there exists a $\delta > 0$, such that for any $\varphi \in C_\tau$, $\|\varphi\| < \delta$ implies $\|x(t, t_0, \varphi, \sigma)\| < \varepsilon$.

(2) It is said to be asymptotical stabilizable under the feedback control u and the switching law σ (ASWUS) if it is SWUS and there exists a $\delta > 0$ such that for any $\varphi \in C_\tau$, $\|\varphi\| < \delta$ implies

$$\lim_{t\to+\infty} x(t, t_0, \varphi, \sigma) = 0.$$

(3) It is said to be globally asymptotical stabilizable under the feedback control u and the switching law σ (GASWUS) if it is ASWUS and for any $\varphi \in C_\tau$, has

$$\lim_{t\to+\infty} x(t, t_0, \varphi, \sigma) = 0.$$

(4) It is said to be exponential stabilizable under the feedback control u and the switching law σ (ESWUS) if there exist constants $M > 0$, $\delta > 0$, $\lambda > 0$, such that for any $\varphi \in C_\tau$, $\|\varphi\| < \delta$ implies

$$|x(t, t_0, \varphi, \sigma)| \le Me^{-\lambda(t-t_0)}, \quad t \ge t_0.$$