Nonlinear Analysis: Hybrid Systems 14 (2014) 114–125

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Stability analysis for stochastic jump systems with time-varying delay

Yun Chen *, Renquan Lu, Hongbo Zou, Anke Xue

Institute of Information and Control, Hangzhou Dianzi University, Hangzhou 310018, China

ARTICLE INFO

Article history:
Received 23 September 2013
Accepted 23 May 2014

Keywords:
Stochastic systems
Markovian jump
Time-varying delay
Stochastic inequality

ABSTRACT

This paper deals with the mean-square asymptotic stability of stochastic Markovian jump systems with time-varying delay. Based on a new stochastic inequality and convex analysis property, some novel stability conditions are presented. In the derivation, the information of the time-varying delay is retained and the estimation of it by the worst-case enlargement is not involved. Some special cases of the systems under consideration are also investigated. Illustrative examples are given to show the effectiveness of the proposed approach.

1. Introduction

In practice, a Markovian jump system can be used to model dynamical systems including random changing structures and parameters, such as repairs of machines in manufacturing systems, component and interconnection failures of fault-tolerant systems, states transitions in maneuvered target tracking, and so on [1]. A Markovian jump system, composed of a finite number of discrete models, is an important class of stochastic systems, and the switching rule among its deterministic modes can be described Markovian character. The theoretical and applied studies on Markovian jump systems have been successfully investigated in the past years, see for example [1–4] and the references therein.

Time delay is frequently intrinsic and inevitable in many real-world dynamical systems, and the existence of delay will degrade the systems' performances and even lead to oscillation and instability. The study of the effects of delay to the systems has attracted a lot of attention in the past decades [5–7]. Experimental paradigms of time-delayed inverted pendulum with implications to human balance control have been examined in [8]. Based on Lyapunov–Krasovskii theory and the linear matrix inequality (LMI) approach, [9] has obtained some delay-dependent criteria for Markovian jump systems with time-varying delay by introducing free matrices. Stability analysis and stabilization problems for discrete-time Markovian jump linear systems with partially known transition probabilities and time-varying delays have been investigated [10]. Most recently, H_∞ control for singular Markovian jump systems with delay has been discussed [11].

On the other hand, lots of practical dynamical systems are affected by random environmental noises describing Brownian motions (or Wiener processes) [12]. The problem of asynchronous $l_2 - l_\infty$ filtering for discrete-time stochastic systems with Markovian jump and random sensor nonlinearities has been handled [13]. Stability and dissipative control for continuous-time Itô-type nonlinear stochastic Markov jump systems have been studied [14,15]. And stochastic systems with time delay have been deeply considered by many recent papers [16–25]. Delay-dependent mean-square stability conditions for stochastic delay systems have been presented based on descriptor model transformation [16,19]. By means of the input–output approach, delay-dependent results for stochastic systems with delay have been proposed [17]. On the basis of the free-weighting matrix technique, some delay-dependent criteria for stochastic time-delay systems have been established recently [18,20,21,23].

* Corresponding author. Tel.: +86 57186919131.
E-mail address: cloudscy@msn.com (Y. Chen).

http://dx.doi.org/10.1016/j.nahs.2014.05.007
1751-570X/

Furthermore, if Markovian systems with time delay are driven by stochastic noises, then stochastic time-delay systems with Markovian jumping parameters are brought forward [26–33]. Mao has handled exponential stability for the stochastic Markovian jump delayed system by the matrix measure technique [26]. Moreover, the free-weighting matrix method has been extended to study stochastic delay systems with Markovian jumping parameters in [27,28,30]. However, when the delay in stochastic systems is time-variant, the term $-\int_{t-h}^{t} x^T(s)Rx(s)ds$ (or the similar ones) has been bounded as $-\int_{t-d(t)}^{t} x^T(s)dsR\int_{t-d(t)}^{t} x(s)ds$ (or the similar ones) by using Jensen's inequality in [20,21,27,28,30]. This enlargement means that the time-varying delay '$d(t)$' is approximated by its upper bound 'h', which results in remarkable conservatism. In order to efficiently overcome the conservatism due to the above-mentioned approximation, the property of the convex analysis method is recalled. [6,7] have shown that the usage of convex analysis can retain the useful information of '$d(t)$' and avoid estimating the delay by its lower and upper bounds. However, the stability analysis for stochastic Markovian jump systems with time-varying delay is still open. The successful application of convex analysis enlightens us to consider stability analysis for stochastic Markovian jump systems with time-varying delay. The above considerations motivate the present paper.

This paper investigates the problem of mean-square asymptotic stability of stochastic Markovian jump systems with time-varying delay. First, an inequality in stochastic scope is presented. Then, combining this stochastic inequality with the property of convex analysis, novel asymptotic stability criteria are established. This method can avoid estimating the time-varying delay '$d(t)$' by its lower and upper bounds, and the information of the delay '$d(t)$' is remained. Moreover, the integral term $-\int_{t-h}^{t} y_i^T(s)Ry_i(s)ds$ is not bounded directly as $-\int_{t-d(t)}^{t} y_i^T(s)Rds\int_{t-d(t)}^{t} y_i(s)ds$ by Jensen's inequality such that our result shows less conservatism than the previous reports. Third, results for some special cases are also provided. Finally, numerical examples are provided to demonstrate the effectiveness of the presented method.

Notations: Throughout this paper, the notations are standard. $\mathrm{tr}\{A\}$ is the trace of matrix A; $\mathrm{diag}\{A_1, \ldots, A_n\}$ represents a block diagonal matrix with diagonal matrix blocks $A_1, \ldots, A_n$; $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ designate the minimal and maximal eigenvalues of a symmetric matrix, respectively. $\mathbf{E}\{\cdot\}$ is the mathematical expectation operator. $\mathbf{Pr}\{\cdot\}$ stands for the probability. $R^{\frac{1}{2}}$ is the square root matrix of the positive definite and symmetric matrix $R > 0$ (i.e., $R^{\frac{1}{2}} = XY^{\frac{1}{2}}X^T$, where X and Y are the eigenvector matrix and the diagonal eigenvalue matrix of matrix R, respectively, with X satisfying $XX^T = I$). $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$ is a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., $\mathscr{F}_0$ contains all P-null sets and the filtration is increasing and right continuous). The symmetric term in a symmetric matrix is denoted as $*$.

2. Problem statement and preliminaries

Consider the following Markovian jump system

$$dx(t) = [A(t, r_t)x(t) + A_1(t, r_t)x(t - d(t)) + f(t, x(t), x(t - d(t)), r_t)]dt + g(t, x(t), x(t - d(t)), r_t)dw(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $A(t, r_t)$ and $A_1(t, r_t)$ are matrix functions of the random jump process $r_t := r(t)$, where $r(t)$ is a finite-state Markovian jump process representing the system mode, i.e., $r(t)$ takes discrete values in a given finite set $\mathbf{S} = \{1, 2, \ldots, \mathscr{N}\}$; $d(t)$ is the time-varying delay satisfying $0 < d(t) \leq h$, $\dot{d}(t) \leq \mu$, where h, μ are scalars; the initial condition of system (1) is given by the real-valued function $\phi(\cdot)$, which is continuously differentiable on $[-h, 0]$; $w(t)$ is a scalar Wiener process (or Brownian motion) defined on the probability space $(\Omega, \mathscr{F}, \mathscr{P})$ satisfying $\mathscr{E}\{dw(t)\} = 0$, $\mathscr{E}\{dw^2(t)\} = dt$. $f(t, x(t), x(t-d(t)), r_t) \in \mathbb{R}^n$ and $g(t, x(t), x(t-d(t)), r_t) \in \mathbb{R}^{n\times m}$ are nonlinear parametric uncertainties and nonlinear stochastic perturbations, respectively. To simplify, for each mode $r_t = i \in \mathbf{S}$, $A(t, r_t)$, $A_1(t, r_t)$, $f(t, x(t), x(t - d(t)), r_t)$ and $g(t, x(t), x(t - d(t)), r_t)$ are abbreviated as A_i, A_{1i}, $f_i(t)$ and $g_i(t)$, respectively.

The transition probability matrix system of (1) is given as

$$\mathbf{Pr}(r_{t+\Delta} = j \mid r_t = i) = \begin{cases} \pi_{ij}\Delta + o(\Delta), & j \neq i \\ 1 + \pi_{ii}\Delta + o(\Delta), & j = i \end{cases}$$

where

$$\Delta > 0, \quad \lim_{\Delta\to 0} \frac{o(\Delta)}{\Delta} = 0, \qquad \pi_{ij} \geq 0, \quad \forall j \neq i$$

is the transition rate from mode i at time t to mode j at time $t + \Delta$, and

$$\pi_{ii} = -\sum_{j=1, j\neq i}^{j=\mathscr{N}} \pi_{ij} < 0.$$

In this paper, two assumptions for system (1) are given as follows.

Assumption 1. Assume that $f_i(t)$ and $g_i(t)$ satisfy

$$|f_i(t)| \leq |F_{1i}x(t)| + |F_{2i}x(t - d(t))| \tag{2}$$

and

$$\mathrm{tr}\{g_i^T(t)g_i(t)\} \leq |G_{1i}x(t)|^2 + |G_{2i}x(t - d(t))|^2 \tag{3}$$

where F_{1i}, F_{2i} and G_{1i}, G_{2i} $(i \in \mathbf{S})$ are known constant matrices with compatible dimensions.