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Stability analysis of a class of nonlinear positive switched systems with delays ☆

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ABSTRACT

This paper addresses the stability problem of delayed nonlinear positive switched systems whose subsystems are all positive. Both discrete-time systems and continuous-time systems are studied. In our analysis, the delays in systems can be unbounded. Two conditions are established to test the local asymptotic stability of the considered systems. The method to compute the domains of attraction is also proposed provided that the system is locally asymptotically stable. When reduced to general nonlinear positive systems, that is, the considered switched system consists of only one mode, an interesting conclusion follows that the proposed nonlinear positive system is locally asymptotically stable for all admissible delays and nonnegative nonlinearities which satisfy an extra condition at the origin, if and only if the system represented by the linear part is asymptotically stable for all admissible delays. Finally, a numerical example is presented to illustrate the obtained results.

1. Introduction

Many systems in the real world can be modeled by dynamic systems with combined continuous and discrete states [1]. Such systems are called switched systems. A switched system consists of a family of (finitely or infinitely many) dynamic subsystems (also called modes) with a rule, called a switching signal, that determines the switching behavior amongst the subsystems. Switched systems have attracted a lot of attention from the control and system theory communities due to their abilities to capture the dynamics of various physical systems [2]. There are still many open and challenging issues remained to be tackled, despite great successes reported during the past several decades [3].

A special class of switched systems is positive switched system (PSS) consisting of a set of positive subsystems. A positive system is one that its state and output are nonnegative whenever the initial condition and input are nonnegative [4–6]. Positive systems have numerous applications in areas such as economics, biology, sociology and communication [7,8]. It is well known that positive systems have many particular and interesting properties. For example, their stability is not affected by delays [9,10] if the delays are bounded. A linear PSS corresponds to a switched system in which each subsystem is itself a linear positive system. It is clear that studying the dynamics of positive switched systems is more challenging than that of general switched systems because, in order to obtain some desirable results, one has to combine the features of positive systems and switched systems [11,12]. As a matter of fact, linear PSSs have gained rapidly increasing attention in recent years. The importance of linear PSSs has been highlighted due to their broad applications in formation flying [13] and systems theory [14]. A wealth of literature is concerned with the issue of stability of linear PSSs [15–18]. It has been proved in [15] that for a linear PSSs consisting of two 2nd order subsystems, it is uniformly asymptotically stable if and only if the system has a common copositive quadratic Lyapunov function. A necessary and sufficient condition has been established in [17]

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for the existence of a common linear copositive Lyapunov function, where the switched system consists of two nth order subsystems. A switched copositive linear Lyapunov function has been proposed in [16], which yields a less conservative stability condition for linear PSSs. Fornasini et al. [19] proposed a series of results on the stability and stabilization properties of discrete-time linear PSSs, exponential stabilizability of continuous-time positive switched systems was investigated in [20], and stabilizability of linear PSSs was studied in [21].

Delays are universal in physical processes and have very significant impacts on their dynamics [22–24]. This fact makes the task of studying the dynamics of delayed PSSs important. Since delayed PSSs form a special class of switched systems, the relevant methods applicable to switched delay systems are also suitable for delayed PSSs. These methods, such as the popular Lyapunov–Krasovskii functional method, however, generally fail to capture the inherent characteristics of delayed PSSs. Therefore, it is necessary to develop new approaches and techniques for analyzing stability of delayed PSSs. The stability issue of delayed PSSs was studied in [25], where it was assumed for discrete-time systems that there exists a common positive vector $\boldsymbol{\lambda}$ such that $\left(\sum_{i=0}^{p} A_{il} - I\right)\boldsymbol{\lambda} \prec 0$ with $\sum_{i=0}^{p} A_{il}$ being the sum of all the system matrices of the lth subsystem. The main idea in [25] is to construct a "cover" that covers all the solution trajectories starting from a specified region. Recently, a "copositive polynomial Lyapunov function" method is proposed for delay-free PSSs [26], which is essentially a kind of higher order quadratic Lyapunov function and cannot be applied to systems with delays.

All the references above mentioned are mainly concerned with linear systems. Practically, most physical systems, chemical systems, biological systems, and man-made systems are nonlinear [27]. Therefore, the emergence of nonlinearity in many situations is inevitable and many researchers have paid their attention to studying the influence caused by nonlinearity. Compared with linear systems, theory of nonlinear systems is less developed due to their inherent complexities. However, many nonlinear systems can be presented as the sum of a linear part and a nonlinear part, enabling it possible to use theorem of linear systems to treat dynamics of nonlinear systems. The present paper will study a class of such systems.

In the context of positive systems, nonlinear positive systems have been studied at least for two decades. Some basic results were established in [28] on the existence and uniqueness of solution for a class of nonlinear symmetric positive systems, Härdin treated the reduction of a class of nonlinear positive systems appearing in biological systems [29], 2D nonlinear positive systems were studied in [30]. Some results to test the positivity of a nonlinear system and the reachability of a nonlinear positive system without delays were proposed in [31]. The nonlinear positive observer design for some positive systems was discussed in [32]. For more literature, see [33] and the cited references therein. However, nonlinear PSSs are seldom investigated. Recently, [34] discussed the D-stability of nonlinear PSSs without delays. On this ground, we will study the stability property of nonlinear PSSs with delays in the paper.

Another motivation lies in: As stated previously, delayed linear PSSs possess very nice stability properties, so it is natural to ask the question: How does a delayed linear PSS behave after adding nonlinearity? And in what condition does the nonlinearity have no influence on system stability, just as the situation that delays have no influence on stability of positive system? This paper will first explore such problems.

This paper focuses on the stability problem of nonlinear PSSs with delays, both discrete- and continuous-time systems are considered. The involved delays may be unbounded in either case. The nonlinear terms considered in this paper are assumed to be differentiable at the zero with differentiation zero. Some criteria are established for verifying the local asymptotic stability properties of nonlinear PSSs with delays, and the methods to compute domain of attraction are given provided that the system is locally asymptotically stable. If the considered switched system consists of only one mode, then we reach an interesting conclusion: A nonlinear positive system is locally asymptotically stable for all admissible delays and nonnegative nonlinearities which satisfy an extra condition at the origin, if and only if the system represented by the linear part is asymptotically stable for all admissible delays.

The rest of this paper is organized as follows. Preliminaries are presented in Section 2, Section 3 studies stability property of discrete-time nonlinear PSSs with delays, and Section 4 treats the continuous case. Section 5 provides an example, and Section 6 concludes this paper.

2. Preliminaries

Nomenclature

$A \succeq 0$ ($\succ 0$)	Matrix A with nonnegative (positive) elements
$A \preceq 0$ ($\prec 0$)	Matrix A with nonpositive (negative) elements
A^{T}	Transpose of matrix A
$\mathbb{R}_{0,+}$	The set of all nonnegative real numbers
$\mathbb{R}^n$ ($\mathbb{R}^n_+$, $\mathbb{R}^n_{0,+}$)	The set of n-dimensional real (positive, nonnegative) vectors
$\mathbb{R}^{n\times m}$	The set of all real matrices of $n \times m$-dimension
$\mathbb{N}$	$\{1, 2, 3, \ldots\}$
$\mathbb{N}_0$	$\{0\} \bigcup \mathbb{N}$
$\underline{p}$	$\{1, 2, \ldots, p\}$, where $p \in \mathbb{N}$
$\lvert a \rvert$	The absolute value of a real number a
$\lVert \boldsymbol{x} \rVert_\infty$	l_∞ norm of vector $\boldsymbol{x} \in \mathbb{R}^n$
$\mathcal{B}_\varepsilon$	$\{\boldsymbol{x} \in \mathbb{R}^n \mid \lVert \boldsymbol{x} \rVert_\infty < \varepsilon\}$