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Adaptive fuzzy backstepping control design for a class of pure-feedback switched nonlinear systems☆

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In this paper, an adaptive fuzzy output tracking control approach is proposed for a class of single input and single output (SISO) uncertain pure-feedback switched nonlinear systems under arbitrary switchings. Fuzzy logic systems are used to identify the unknown nonlinear system. Under the framework of the backstepping control design and fuzzy adaptive control, a new adaptive fuzzy output tracking control method is developed. It is proved that the proposed control approach can guarantee that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB) and the tracking error remains an adjustable neighborhood of the origin. A numerical example is provided to illustrate the effectiveness of the proposed approach.

1. Introduction

Switched systems are dynamical systems consisting of a collection of continuous-time subsystems and a switching rule that orchestrates the switching among them [1–4]. In the last decades, switched systems have attracted more and more attention due to their significance in the modeling of many engineering applications, such as chemical processes, robot manipulators and power systems. So far, many remarkable achievements about the stability analysis and synthesis have been made in the field of switched systems commonly by two kinds of methods: one is to find a common Lyapunov function to ensure stability of the switched systems under arbitrary switching laws; the other is to use a multiple Lyapunov functional technique to stabilize the switched systems under some designed switching laws, see, e.g., [5–11] and the references therein.

Backstepping control design is a recursive design methodology [12]. With this methodology the construction of both feedback control laws and associated Lyapunov functions is systematic. Recently, some backstepping control design methods have been proposed for several classes of switched nonlinear systems [13–15]. Three state feedback control approaches in [13–15] have investigated based on the common Lyapunov function method for a class of switched nonlinear systems, but the nonlinear functions of controlled systems in [13–15] are required to be known functions.

It has been proven that the fuzzy logic systems (FLS) and neural networks (NN) can approximate arbitrary nonlinear continuous function to a given accuracy on a closed set [16,17]. Therefore, the unknown nonlinear functions of switched nonlinear systems can be approximated by FLS or NN. Based on this idea, [18] has proposed an adaptive neural network control for a class of switched nonlinear systems with completed unknown nonlinear functions, but this approach needs

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construct switching law, that is the switching in [18] is not arbitrary. In [19], a novel adaptive neural network control method has been presented for a class of switched nonlinear systems with switching jumps and uncertainties in both system models and switching signals, the condition about switching signal's dwell-time property need to be satisfied in [19]. In addition, the controlled systems in [18,19] are strict-feedback systems, not pure-feedback systems. As stated in [20,21], unlike a nonlinear system in strict-feedback form, a nonlinear system in pure-feedback form has no affine appearance of the state variables to be used as virtual controls and the actual control in the backstepping recursive design technique, which makes the control design and the stability analysis of the closed-loop system more difficult and challenging. In recent years, several adaptive NN or fuzzy backstepping control approaches have been developed for nonlinear systems in pure-feedback form [20–28]. However, these control approaches in [20–28] are used to control non-switched nonlinear systems and cannot be applied to switched nonlinear systems.

Motivated by the above observations, an adaptive fuzzy output tracking control approach is presented for a class of uncertain pure-feedback switched nonlinear systems with completed unknown nonlinear functions, under arbitrary switchings. In the control design, fuzzy logic systems are used to approximate the unknown nonlinear system functions, and an adaptive fuzzy control method has been developed. It is shown that the proposed adaptive controller can guarantee that all the signals in the closed-loop systems remain SGUUB, and the tracking error converges to a small neighborhood of the origin.

The main contributions of this paper can be summarized as follows:

(1) By using fuzzy logic systems to approximate the unknown nonlinear system functions, the proposed control method can solve the tracking control problem of nonlinear switched systems with unknown nonlinear functions, while, Refs. [13,15] required the system functions to be known.

(2) The considered nonlinear switched systems are pure feedback form, and the controlled systems in [18,19] are strict feedback form. The control design and stability analysis of the closed-loop system in this paper are more difficult compared with Refs. [18,19].

(3) Both problems "curse of dimensionality" and "explosion of complexity" of nonlinear switched systems are avoided simultaneously, which leads to a much simpler controller with less computational burden.

2. Problem formulations and preliminaries

2.1. Problem formulation

Consider the following a class of pure-feedback nonlinear switched systems:

$$\begin{cases} \dot{x}_i = F_{\sigma(t),i}(\bar{x}_i, x_{i+1}), \quad 1 \le i \le n-1 \\ \dot{x}_n = F_{\sigma(t),n}(\bar{x}_n, u) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^{\mathrm{T}} \in R^i$, $i = 1, \ldots, n$, is the state vector of the system. $u \in R$ and $y \in R$ are the system control input and output, respectively. $\sigma(t) : [0, \infty) \to \Xi \overset{def}{=} \{1, 2, \ldots, N\}$ is a piecewise constant function called switching signal (or law), which takes values in the compact set Ξ. If $\sigma(t) = p$, then we say the pth subsystem is active and the remaining subsystems are inactive. $F_{p,i}(\cdot)(p \in \Xi)$, $i = 1, \ldots, n$, are unknown smooth functions.

The aim of this paper is to develop an adaptive fuzzy control scheme for a class of pure-feedback nonlinear switched systems such that all the signals of the closed-loop system are bounded and the system output y of system (1) follows any given bounded desired output signal y_r.

In order to get explicit control variable or virtual ones, one can express $F_{p,i}(\bar{x}_i, x_{i+1})$ and $F_{p,n}(\bar{x}_n, u)$ with the help of mean value theorem as follows:

$$F_{p,i}(\bar{x}_i, x_{i+1}) = F_{p,i}(\bar{x}_i, x_{i+1}^0) + \left.\frac{\partial F_{p,i}(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}\right|_{x_{i+1}=x_{i+1}^{\xi_{p,i}}} \times (x_{i+1} - x_{i+1}^0) \quad (1 \le i \le n+1) \tag{2}$$

$$F_{p,n}(\bar{x}_n, u) = F_{p,n}(\bar{x}_n, u^0) + \left.\frac{\partial F_{p,n}(\bar{x}_n, u)}{\partial u}\right|_{u=u^{\xi_{p,n}}} \times (u - u^0) \tag{3}$$

in which smooth function $F_i(\cdot)$ is explicitly analyzed between $F_{p,i}(\bar{x}_i, x_{i+1})$ and $F_{p,i}(\bar{x}_i, x_{i+1}^0)$. $x_{i+1} = x_{i+1}^{\xi_{p,i}}$ is some point between x_{i+1} and x_{i+1}^0; $\xi_{p,i}$ is some constant satisfying $0 < \xi_{p,i} < 1$ and $1 \le i \le n-1$. For convenience, denote $x_{n+1} = u$.

Further, by choosing $x_{i+1}^0 = 0$, (2) and (3) are expressed as

$$F_{p,i}(\bar{x}_i, x_{i+1}) = F_{p,i}(\bar{x}_i, 0) + \left.\frac{\partial F_{p,i}(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}\right|_{x_{i+1}=x_{i+1}^{\xi_{p,i}}} \times (x_{i+1} - 0) \quad (1 \le i \le n-1) \tag{4}$$

$$F_{p,n}(\bar{x}_n, u) = F_{p,n}(\bar{x}_n, 0) + \left.\frac{\partial F_{p,n}(\bar{x}_n, u)}{\partial u}\right|_{u=u^{\xi_{p,n}}} \times (u - 0). \tag{5}$$