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Disturbance tolerance and rejection of discrete switched systems with time-varying delay and saturating actuator

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ABSTRACT

This paper is concerned with the problems of disturbance tolerance and rejection of discrete switched systems with time-varying delay and saturating actuator. Using the switched Lyapunov function approach, a sufficient condition for the existence of a state feedback controller is proposed such that the disturbance tolerance capability of the closed-loop system is ensured. By solving a convex optimization problem with linear matrix inequality (LMI) constraints, the maximal disturbance tolerance is estimated. In addition, the problem of disturbance rejection of the closed-loop system is solved. Two examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

A switched system, a typical hybrid system, consists of several subsystems and a switching signal. Each subsystem could be recognized as a separated one since the dynamic behavior of the switched system is represented by the activation between those subsystems. Switched systems have various applications in many fields, such as the engine control systems [1], car-pendulum systems [2], robot control systems [3], network control systems [4–6], computer disk control systems [7]. There have been a lot of methodologies and techniques for the study of switched systems, such as the common Lyapunov function method, the switched Lyapunov function method and the average dwell time approach. Numerous researches have focused on the stability analysis and controller design of switched systems [8–15].

It is well known that time delay appears frequently in practical engineering and may deteriorate the stability and performance of the system. Thus, time delay systems have received more and more attention, and many results have been reported [12–24]. Some of them are concerned with switched systems with time-varying delay [12–16]. For example, [16] dealt with the problem of robust state-feedback control for uncertain discrete-time switched systems with mode-dependent time-varying delays.

On the other hand, actuator saturation is also a common phenomenon in practice and may lead to the degradation of the stability and performance of a given system. A growing number of scholars have dedicated themselves to studying the stabilization of the system with actuator saturation [25–29]. At the same time, lots of approaches have been worked out to solve this problem. For example, [30] investigated the stability analysis and anti-windup design problems for a class of discrete-time switched linear systems with time-varying norm-bounded uncertainties and saturating actuators by using the switched Lyapunov function approach.

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Since disturbances are unavoidable in practical application, it is worth investigating the tolerance and rejection of the system to disturbance. L_2-gain performance index is one of the most important indices to describe the relation between the disturbances and outputs. For example, [31] analyzed the disturbance tolerance and rejection of discrete-time stochastic systems with actuator saturation. [32] considered the problem of H_∞ model approximation for discrete-time Takagi–Sugeno (T–S) fuzzy time delay system. However, to the best of our knowledge, the problem of disturbance tolerance and rejection of discrete-time switched systems with time-varying delay and actuator saturation has not been fully investigated, which motivates us for this study.

In this paper, we focus our interest on disturbance tolerance and rejection of discrete switched systems with time-varying delay and actuator saturation. The switched Lyapunov function approach is utilized for the controller design. The main contributions of this paper are threefold: (1) The disturbance tolerance of the system under consideration is firstly considered, and we present sufficient conditions under which the trajectories of the given system starting from a bounded set remain bounded for any bounded disturbances; (2) The maximal disturbance tolerance is estimated by solving a convex optimization problem with linear matrix inequality constraints; (3) The disturbance rejection capability of the system is also analyzed.

The remainder of the paper is organized as follows. In Section 2, problem formulation and some necessary lemmas are given. In Sections 3 and 4, the disturbance tolerance and rejection capabilities are investigated. The effectiveness of the proposed approach is illustrated by two examples in Section 5. Conclusions are in Section 6.

Notations: The notations used throughout the paper are standard. R stands for the set of real numbers. R^n denotes the set of real valued vectors. $R^{m\times n}$ represents the set of real $m \times n$ matrices. The notation F^T represents the transpose of the matrix F. For a matrix $F \in R^{m\times n}$, $L(F) := \{x(k) \in R^n : |Fx(k)| \le 1\}$ is the linear region of the saturation function $sat(Fx(k))$. For a symmetric and positive definite matrix $P \in R^{n\times n}$ and a positive scalar ρ, $\varepsilon(P, \rho) := \{x(k) \in R^n : x^T(k)Px(k) \le \rho\}$ denotes the ellipsoid. Z_0^+ denotes the set of all nonnegative integers. $\lambda_{\max}(P)$ denotes the maximum eigenvalue of the matrix P. I represents the identity matrix with an appropriate dimension, $\mathrm{diag}\{a_i\}$ denotes the diagonal matrix with the diagonal elements $a_i,\ i = 1, 2, \ldots, n$. X^{-1} denotes the inverse of X. The asterisk $*$ in a matrix is used to denote the term that is induced by symmetry.

2. Problem formulation and preliminaries

Consider the following discrete-time switched system with time-varying delay and saturating actuator:

$$x(k+1) = A_{\sigma(k)}x(k) + A_{d\sigma(k)}x(k-d(k)) + B_{\sigma(k)}sat(u(k)) + E_{\sigma(k)}w(k), \tag{1a}$$

$$z(k) = C_{\sigma(k)}x(k) + C_{d\sigma(k)}x(k-d(k)) + L_{\sigma(k)}w(k), \tag{1b}$$

$$x(k_0+\theta) = \varphi(\theta), \quad \theta = -d_2, -d_2+1, \ldots, 0, \tag{1c}$$

where $x(k) \in R^n$, $u(k) \in R^m$, $w(k) \in R^h$ and $z(k) \in R^p$ are the state vector, control input, disturbance and output, respectively. $d(k)$ is a time-varying delay satisfying $0 \le d_1 \le d(k) \le d_2$. $k_0 = 0$ is the initial time. The function $\sigma(k) : Z_0^+ \to \underline{N} := \{1, \ldots, N\}$ is the switching signal with N being the number of subsystems. A_i, A_{di}, B_i, E_i, C_i, C_{di} and L_i, $\forall i \in \underline{N}$, are known constant matrices. $sat : R^m \to R^m$ is a saturating function which is defined as

$$sat(u(k)) = [sat(u_1(k))\ sat(u_2(k)) \ldots sat(u_m(k))]^T,$$

where, $\forall \tau \in [1, m]$,

$$sat(u_\tau(k)) = \begin{cases} -1 & u_\tau(k) < -1, \\ u_\tau & |u_\tau(k)| \le 1, \\ 1 & u_\tau(k) > 1. \end{cases}$$

Let D be the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. We can easily find that there are 2^m elements in D. Let these elements be defined as $D_s,\ s \in [1,\ 2^m]$, and denote $D_s^- = I - D_s$. It is easy to see that $D_s^- \in D$. For the sake of simplicity, we define $w_\alpha^2 := \left\{w : R^+ \to R^q : \sum_{k=0}^{+\infty} w^T(k)w(k) \le \alpha\right\}$.

Applying the controller $u(k) = F_{\sigma(k)}x(k)$ to system (1) leads to the following closed-loop system

$$x(k+1) = A_{\sigma(k)}x(k) + A_{d\sigma(k)}x(k-d(k)) + B_{\sigma(k)}sat(F_{\sigma(k)}x(k)) + E_{\sigma(k)}w(k), \tag{2a}$$

$$z(k) = C_{\sigma(k)}x(k) + C_{d\sigma(k)}x(k-d(k)) + L_{\sigma(k)}w(k). \tag{2b}$$

The following lemmas will be essential for our later development.

Lemma 1 ([33]). *Let matrices $F \in R^{m\times n}$, $H \in R^{m\times n}$, and suppose that $x(k) \in L(H)$, then $sat(Fx(k)) \in co\{D_sFx(k) + D_s^-Hx(k) : s \in [1,\ 2^m]\}$, where $co\{\cdot\}$ stands for the convex hull.*