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Stabilization of switched systems via optimal control

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ABSTRACT

We consider switched systems composed of linear time invariant unstable dynamics and we deal with the problem of computing an appropriate switching law such that the controlled system is globally asymptotically stable. On the basis of our previous results in this framework, we first present a method to design a feedback control law that minimizes a linear quadratic (LQ) performance index when an infinite number of switches are allowed and at least one dynamics is stable. Then, we show how this approach can be useful when dealing with the stabilization problem of switched systems characterized by unstable dynamics, by applying the proposed procedure to a "dummy" system, augmented with a stable dynamics. If the system with unstable dynamics is globally exponentially stabilizable, then our method provides the feedback control law that minimizes the chosen quadratic performance index, and that guarantees the closed loop system to be globally asymptotically stable.

1. Introduction

In this paper we show how it is possible to design a stabilizing law $i(t)$ for linear time-invariant (LTI) switched systems

$$\dot{x}(t) = A_{i(t)}x(t), \quad i(t) \in \mathcal{S} = \{1, \ldots, s\}, \tag{1}$$

where for all $i \in \mathcal{S}$, A_i are unstable.

Note that here all subsystems are autonomous, i.e., the only control action is the switching function $i(t)$.

The proposed procedure is based on the solution of an optimal control problem for the above switched system.

In [1] we have presented a technique to solve an optimal control problem with quadratic performance index for systems of the form (1) assuming that a finite number of switches N is allowed. The solution takes the form of a state feedback law, i.e., the optimal value of $i(t^+)$ can be chosen as a function of the current continuous state $x(t)$, of the current dynamics $i = i(t)$ and of the number of remaining switches k. The feedback law is described by a set of partitions of the state space $\mathcal{C}_{i,k}$ that, for a given current dynamics i and for a given number of remaining switches k, assigns to each continuous state $x(t)$ the optimal value of $i(t^+)$.

In [1] we dealt with the case of finite N, hence we assumed that at least one dynamics is stable to ensure that the considered optimal control problem has a finite cost. In such a case the system is trivially stabilizable: just use the stable dynamics. In this paper, on the contrary, we consider the case in which all dynamics are unstable and thus an infinite number of switches are required to stabilize the system.

Our stabilization procedure is based on two ideas.

Firstly following [2], we show that an optimal control law for infinite switches can be easily computed solving an optimal control problem for a finite number N of switches, provided N is large enough. We also show that in this case the optimal

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control law is still given by a partition of the state space $\mathcal{C}_\infty$ that, however, does not depend on the current location $i(t)$ and on the number of remaining switches (that is obviously infinite).

Secondly, to relax the assumption that at least one dynamics is stable, we extend system (1) by adding an arbitrary dummy stable dynamics $s + 1$. We also show that if the cost associated to dynamics $s + 1$ is sufficiently large, then the system is stabilizable if and only if no region of partition $\mathcal{C}_\infty$ is associated to dynamics $s + 1$.

It is important to stress that the presented stabilization procedure is not necessarily related to the optimal control technique presented in [1]. In fact, it can be applied in tandem to any procedure that solves an optimal control problem providing a feedback law with a finite number of switches, such as the one presented by Shaikh and Caines in [3].

Moreover we observe that using an optimal control approach to stabilize a switched system has already been investigated by other authors. See e.g. the recent survey by Margaliot [4].

Other interesting approaches to the optimal control of switched systems have been proposed by Xu and Antsaklis [5], by Egerstedt, Wardi et al. [6,7], and by Axelsson et al. [8]. However such procedures cannot be directly applied within this framework in their present form because they do not determine a state feedback control law.

The main advantage of the stabilizing approach we present is that it provides a systematic procedure to compute a stabilizing law when it does exist. In fact, although there is a rich literature on stability analysis of hybrid systems, there are very few results on the design of stabilizing laws. The literature in this area is surveyed in the next subsection.

Finally, we observe that a disadvantage of the optimal control approach [1] we use, that is common to several other approaches (see e.g. the paper by Hedlund and Rantzer [9]), is that it requires a discretization of the state space. For large dimensional systems this may be computationally burdensome. Moreover, the discretization may also affect the optimality of the solution. However, here we are concerned with stability, thus this problem is less important. This issue has been extensively addressed in [1] and it is not discussed in this paper. Note, however, that as stated above, the use of the procedure in [1] is not a strict requirement for the presented stabilization procedure.

1.1. Literature review

Many papers on stability and stabilizability of switched linear systems have been published in the last two decades. In this section we provide a short overview of the most important contributions in this topic with particular attention to those results that are closely related to this paper. In particular, we focus on autonomous switched linear systems, i.e., systems with no control input [10,11]. For a more exhaustive survey on these topics we address the reader to the recent work by Lin and Antsaklis [12].

The first problem that has been investigated in this framework is that of stability under arbitrary switching. Solutions to this problem have been proposed based on common quadratic Lyapunov functions (CQLF) [11,13–17] or on switched quadratic Lyapunov functions [18]. In [12] some necessary and sufficient conditions are given. In particular, it is shown that the asymptotic stability problem for switched linear systems with arbitrary switching is equivalent to the robust asymptotic stability problem for polytopic uncertain linear time-variant systems, thus allowing us to use a series of conditions that exist in this framework [19].

Several results have also been proposed in the literature under the assumption that the switching signals satisfy certain constraints, namely under restricted switching. In many applications this is definitely realistic and it is unnecessarily conservative to impose that stability should hold under arbitrary switching. Such restrictions may either arise in the order in which current modes can be active, or in the minimum time that each mode should remain active once it has been activated. Many contributions in this framework are based on the multiple Lyapunov functions (MLF). See, e.g., [10,20] and a series of references therein.

Another fundamental issue, related to stability analysis, is the stabilization problem, i.e., the synthesis of a stabilizing law. A really rich literature on this has been produced in the last few years. See, e.g. [10,18,21] just to mention a few. However, the main limitation of such approaches is that they only give sufficient conditions for the existence of a stabilizing law.

Necessary and sufficient conditions are given in [22] in the case of a switched system commuting between two subsystems, when the performance index under consideration is the quadratic stability of the switched systems. The main feature of this property is that it requires for uncertain systems a quadratic Lyapunov function which guarantees asymptotic stability for all uncertainties under consideration, and is thus a kind of robust stability with very good properties, yet usually needs more restrictive conditions [23]. Iterative algorithms for constructing such common Lyapunov functions can be found in [24].

Antsaklis et al. in [25], using a geometric approach, were able to obtain necessary and sufficient conditions for asymptotic stabilizability of switched systems with an arbitrarily large number of *second-order* LTI unstable systems. When the switched system is asymptotically stabilizable, they also provide an approach to compute a stabilizing law.

Finally, Lin and Antsaklis [26] derived a necessary and sufficient condition for the existence of a switching control law (in static feedback form) for asymptotic stabilization of continuous-time switched linear systems.

1.2. Paper structure

The paper is structured as follows. In Section 2 the problem statement is formally introduced and some preliminary results used in the rest of the paper are provided. Section 3 discusses the optimal control problem in the case of a finite