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Decentralized stability for switched nonlinear large-scale systems with interval time-varying delays in interconnections

Nguyen T. Thanh [a], Vu N. Phat [b,*]

[a] *Department of Mathematics, Hanoi University of Mining and Geology, Hanoi, Viet Nam*

[b] *Institute of Mathematics, VAST, 18 Hoang Quoc Viet Road, Hanoi 10307, Viet Nam*

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A B S T R A C T

In this paper, the problem of decentralized stability of switched nonlinear large-scale systems with time-varying delays in interconnections is studied. The time delays are assumed to be any continuous functions belonging to a given interval. By constructing a set of new Lyapunov–Krasovskii functionals, which are mainly based on the information of the lower and upper delay bounds, a new delay-dependent sufficient condition for designing switching law of exponential stability is established in terms of linear matrix inequalities (LMIs). The developed method using new inequalities for lower bounding cross terms eliminate the need for overbounding and provide larger values of the admissible delay bound. Numerical examples are given to illustrate the effectiveness of the new theory.

1. Introduction

Switching systems belong to an important class of hybrid systems, which are described by a family of differential equations together with specified rules to switch between them. A switching system can be represented by a differential equation of the form

$$\dot{x}(t) = f_\sigma(t, x(t)), \quad t \geq 0,$$

where $\{f_\sigma(.,.) : \sigma \in \mathcal{I}\}$ is a family of functions parameterized by some index set $\mathcal{I}$, which is typically a finite set, and $\sigma(.)$, which depends on the system state at each time, is the switching rule/signal determining a switching sequence for the given system. Switching systems arise in many practical processes that cannot be described by exclusively continuous or exclusively discrete models, such as manufacturing, communication networks, automotive engineering control, chemical processes [1–3]. During the last decades, the stability problem of switched linear time-delay systems has attracted a lot of attention [4–11].

On the other hand, there has been a considerable research interest in large-scale interconnected systems. A typical large-scale interconnected system such as a power grid consists of many subsystems and individual elements connected together to form a large, complex network capable of generating, transmitting and distributing electrical energy over a large geographical area. In general, a large-scale system can be characterized by a large number of variables representing the system, a strong interaction between subsystem variables, and a complex interaction between subsystems. The problem of decentralized control of large-scale interconnected dynamical systems has been receiving considerable attention, because there

* Corresponding author.
E-mail address: vnphat@math.ac.vn (V.N. Phat).

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are a large number of large-scale interconnected dynamical systems in many practical control problems, e.g., transportation systems, power systems, communication systems, economic systems, social systems, and so on. [12–15]. The operation of large-scale interconnected systems requires the ability to monitor and stabilize in the face of uncertainties, disturbances, failures and attacks through the utilization of internal system states. However, even with the assumption that all the state variables are available for feedback control, the task of effective controlling a large-scale interconnected system using a global (centralized) state feedback controller is still not easy as there is a necessary requirement for information transfer between the subsystems [16–19].

The majority of the previous works treated asymptotic stability for switching linear time delay systems under arbitrary switching signal law. The exponential stability problem was considered in [20] for switching linear systems with impulsive effects by using the matrix measure concept, and in [21] for nonholonomic chained systems with strongly nonlinear input/state driven disturbances and drifts. Some extending results of [22–31] for switched systems with time-varying delays, however, the time delays are assumed to be differentiable and the switching rule was constructed on the solutions of a set of LMIs. To the best of our knowledge, there has been no investigation on the exponential stability of switched nonlinear large-scale systems with time-varying delays interacted between subsystems. In fact, this problem is difficult to solve; particularly, when the time-varying delays are interval, non-differentiable and the output is subjected to such time-varying delay functions. The time delay is assumed to be any continuous function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available, but the delay function is bounded but not necessary to be differentiable. This allows the time-delay to be a fast time-varying function and the lower bound is not restricted to being zero. It is clear that the application of any memoryless feedback controller to such time-delay systems would lead to closed-loop systems with interval time-varying delays. The difficulties then arise when one attempts to derive exponential stability conditions. Indeed, existing Lyapunov–Krasovskii functionals and their associated results in [9,10,15,18,19,22,23] cannot be applied to solve the problem posed in this paper as they would either fail to cope with the non-differentiability aspects of the delays, or lead to very complex matrix inequality conditions and any technique such as matrix computation or transformation of variables fails to extract the parameters of the memoryless feedback controllers. This has motivated our research.

In this paper, we consider a class of large-scale nonlinear systems with interval time-varying delays in interconnections. Compared to the existing results, our result has its own advantages. (i) Stability analysis of previous papers reveals some restrictions: The time delay was proposed to be either time-invariant interconnected or the lower delay bound is restricted to being zero, or the time delay function should be differential and its derivative is bounded. In our result, the above restricted conditions are removed for the large-scale systems. In addition the time delay is assumed to be any continuous function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available, but the delay function is bounded but not necessary to be differentiable. This allows the time-delay to be a fast time-varying function and the lower bound is not restricted to being zero. (ii) The developed method using new inequalities for lower bounding cross terms eliminate the need for over bounding and provide larger values of the admissible delay bound. We propose a set of new Lyapunov–Krasovskii functionals, which are mainly based on the information of the lower and upper delay bounds. (iii) The conditions will be presented in terms of the solution of LMIs, that can be solved numerically in an efficient manner by using standard computational algorithms [32]. (iv) A simple geometric design is employed to find the switching law and our approach allows to compute simultaneously the two bounds that characterize the exponential stability rate of the solution.

The paper is organized as follows. Section 2 presents definitions and some well-known technical propositions needed for the proof of the main results. Mail result for designing switching rule of exponential stability of the system is presented in Section 3. Numerical examples showing the effectiveness of the obtained results are given in Section 4. The paper ends with conclusions and cited references.

2. Preliminaries

The following notations will be used throughout this paper, R^+ denotes the set of all real-negative numbers; R^n denotes the n-dimensional space with the scalar product $(.,.)$ and the vector norm $\|\cdot\|$; $R^{n\times r}$ denotes the space of all matrices of $(n \times r)$-dimension. A^T denotes the transpose of A; a matrix A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\lambda : \lambda \in \lambda(A)\}$; $\lambda_A = \lambda_{\max}(A^TA)$; $C^1([a,b],R^n)$ denotes the set of all R^n-valued differentiable functions on $[a,b]$; $L_2([0,\infty],R^r)$ stands for the set of all square-integrable R^r-valued functions on $[0,\infty]$. The symmetric terms in a matrix are denoted by $*$. Matrix A is semi-positive definite ($A \geq 0$) if $(Ax,x) \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $(Ax,x) > 0$ for all $x \neq 0$; $A \geq B$ means $A - B \geq 0$. The segment of the trajectory $x(t)$ is denoted by $x_t = \{x(t+s) : s \in [-\tau,0]\}$ with its norm

$$\|x_t\| = \sup_{s\in[-\tau,0]} \|x(t+s)\|.$$

Consider a class of nonlinear switched large-scale systems with time-varying delays composed of N interconnected subsystems $\Sigma_i,\ i = 1,2,\dots,N$ of the form:

$$\begin{cases} \dot{x}_i(t) = A_i^{\sigma_i}x_i(t) + \displaystyle\sum_{j\neq i,j=1}^{N} A_{ij}^{\sigma_i}x_j(t-h_{ij}(t)) + f_i^{\sigma_i}(t,x_i(t),\{x_j(t-h_{ij}(t))\}_{j=1,j\neq i}^{N}), \\ x_i(t) = \varphi_i(t), \quad \forall t \in [-h_2,0], \end{cases} \tag{2.1}$$