Nonlinear Analysis: Hybrid Systems 11 (2014) 57–70

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Hybrid control of impulsive systems with distributed delays

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ARTICLE INFO

Article history:
Received 19 February 2013
Accepted 15 June 2013

Keywords:
Hybrid systems
Switched systems
Distributed delays
State-dependent switching
Stabilization
Switching control
Impulsive control
Nonlinear perturbations

ABSTRACT

This paper investigates the stabilization of a class of nonlinear systems with distributed delays using impulsive control and switching control. Stabilizing impulsive forces as well as destabilizing disturbance impulses are considered. Verifiable sufficient conditions are established which guarantee the asymptotic or exponential stability of switched and impulsive systems with distributed delays. Results are found for when the impulses are applied at pre-specified times or at the switching instances. The criteria found are based on a special type of state-dependent switching rule which partitions the state space into stabilizing subregions. The main results are proved using a common Lyapunov functional.

1. Introduction

There has been an increased interest recently in studying switched systems, which are a type of hybrid system, as these systems have important applications in many different areas such as mechanical systems, the automotive industry, air traffic control, robotics, intelligent vehicle/highway systems, integrated circuit design, multimedia, manufacturing, power electronics, chaos generators, automotive engine management, high-level flexible manufacturing systems, job scheduling, interconnected power systems, and chemical processes [1–4]. Switched systems, which are governed by a combination of continuous/discrete dynamics and logic-based switching, can model a natural system whose dynamics change abruptly in time or a continuous system being stabilized by switching controllers [1,5]. The latter context is the focus of this paper, as using switching controllers has been shown to help stabilize unstable continuous systems where traditional control may be inadequate [3,6]. It has also been noted that most of the switched systems literature is concerned with continuous/discrete switched systems which fail to properly model physical systems that exhibit impulsive effects [3]. This is an important problem to study since the combination of switching control and impulsive control can increase the desired performance of a system [6]. For examples of the switched systems literature, including results on the stability of impulsive switched systems with delays, see [1,4,5,7–12] and the references therein.

Some work has been done in the literature on the stabilization of unstable continuous systems using switching control. Lin and Antsaklis detailed some results on the switching stabilization of linear systems in the survey paper [13]. Wicks et al. first constructed a special state-dependent switching rule in [14] which guarantees the stabilization of a linear switched system. This type of state-dependent switching rule has been studied further by Kim et al. [15] for linear switched systems with discrete delay. Phat and Ratchagit [16] analyzed the stability of a linear discrete-time system with time-varying discrete delay under a state-dependent switching rule. Gao et al. [17] extended the state-dependent switching rule by considering

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1751-570X/$ – see front matter
http://dx.doi.org/10.1016/j.nahs.2013.06.003

linear switched systems with discrete delay and distributed delays. In [18], Li et al. investigated a linear switched system with mixed delays, including switched delays, and a state-dependent switching rule. In [19], Hien and Phat considered applying switching proportional, delay, and integral controllers to a linear switched system with distributed delays. The authors established a state-dependent switching rule which ensures stabilization under feedback control. Phat et al. [20] analyzed the state-dependent switching rule when applied to a linear switched system with time-varying discrete delay and nonlinear perturbations. In [21], Hien et al. proved stability of a linear switched system with time delay and uncertainty under a state-dependent switching rule. The stabilization of a system with mode-dependent time-varying delays via state-dependent switching was studied in [22] by Liu. In [23], Liu et al. investigated the stability of a switched system of nonlinear ordinary differential equations under a state-dependent switching rule.

The reports discussed above do not incorporate impulsive effects into the system. Hence, the main objective of this paper is to investigate the stabilization of an unstable continuous system with distributed delays using switching and impulsive control. In particular, two cases are considered for the impulsive control: impulses applied at pre-specified times and impulses applied at the switching instants. The first case was studied in [24] and the second case in [25], however, these reports did not include discrete and distributed delays. Further, the analysis in this paper is done in a general way so that the results also apply to systems which exhibit impulsive perturbations or disturbances. Therefore, the main contributions of this paper are to extend the current literature by providing sufficient conditions for the asymptotic or exponential stability of switched and impulsive systems with nonlinear perturbations and distributed delays under state-dependent switching.

The rest of this paper is organized as follows: some preliminaries are given in Section 2. In Section 3, the control problem is formulated in the context of switching and impulsive control. The main results of the paper are presented and proved in Section 4. First, impulsive forces at pre-specified times are considered, then impulsive effects at each switching instant are investigated. Simulations are presented in Section 5 to illustrate the results found. Finally, conclusions and future directions are given in Section 6.

2. Preliminaries

Let $\mathbb{R}^n$ denote the Euclidean space of n-dimensions with Euclidean norm $\|\cdot\|$, let $\mathbb{R}_+$ denote the set of non-negative real numbers, and let $\lambda_{\max}[Q]$ ($\lambda_{\min}[Q]$) denote the maximum (minimum) eigenvalue of a symmetric matrix Q, respectively. Define $x(t^+) = \lim_{a\to 0^+} x(t+a)$ and $x(t^-) = \lim_{a\to 0^+} x(t-a)$. Assume that a and b are constants satisfying $a < b$ and assume that $S \subset \mathbb{R}^n$, then define the following classes of piecewise continuous functions [26]:

$$PC([a,b],S) = \{x : [a,b] \to S \mid x(t) = x(t^+) \text{ for all } t \in [a,b);\ x(t^-) \text{ exists in } S \text{ for all } t \in (a,b];$$
$$x(t^-) = x(t) \text{ for all but at most a finite number of points } t \in (a,b]\}$$
$$PC([a,b),S) = \{x : [a,b) \to S \mid x(t) = x(t^+) \text{ for all } t \in [a,b);\ x(t^-) \text{ exists in } S \text{ for all } t \in (a,b);$$
$$x(t^-) = x(t) \text{ for all but at most a finite number of points } t \in (a,b)\}$$
$$PC([a,\infty),S) = \{x : [a,\infty] \to S \mid \text{for all } c > a,\ x|_{[a,c]} \in PC([a,c],S)\}.$$

Given a constant $h^* > 0$, equip the linear space $PC([-h^*,0],\mathbb{R}^n)$ with the norm $\|\psi\|_{h^*} = \sup_{-h^*\le\theta\le 0}\|\psi(\theta)\|$. Then $(PC([-h^*,0],\mathbb{R}^n), \|\cdot\|_{h^*})$ is a Banach space.

Define classes of Lyapunov functions and functionals as follows.

Definition 1 (*[26]*)**.** A function $V : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_+$ is said to belong to the class ν_0 if

(i) V is continuous in each of the sets $[t_{k-1}, t_k) \times \mathbb{R}^n$ and for each $x, y \in \mathbb{R}^n$, $t \in [t_{k-1}, t_k)$, $k = 1, 2, \ldots,$

$$\lim_{(t,y)\to(t_k^-,x)} V(t,y) = V(t_k^-, x)$$

exists;

(ii) $V(t,x)$ is locally Lipschitzian in all $x \in \mathbb{R}^n$, and $V(t,0) \equiv 0$ for all $t \ge t_0$.

Definition 2 (*[26]*)**.** A functional $V : \mathbb{R}_+ \times PC([-h^*,0],\mathbb{R}^n) \to \mathbb{R}_+$ is said to belong to the class $\nu_0^*(\cdot)$ if

(i) V is continuous on $[t_{k-1}, t_k) \times PC([-h^*,0],\mathbb{R}^n)$ and for all $\psi, \phi \in PC([-h^*,0],\mathbb{R}^n)$, and $k = 1, 2, \ldots,$

$$\lim_{(t,\psi)\to(t_k^-,\phi)} V(t,\psi) = V(t_k^-, \phi)$$

exists;

(ii) $V(t,\psi)$ is locally Lipschitzian in ψ in each compact set in $PC([-h^*,0],\mathbb{R}^n)$, and $V(t,0) \equiv 0$ for all $t \ge t_0$;

(iii) for any $x \in PC([t_0 - h^*, \infty), \mathbb{R}^n)$, $V(t, x_t)$ is continuous for $t \ge t_0$.