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Stabilization of switched systems with polytopic uncertainties via composite quadratic functions

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ABSTRACT

This paper studied the stabilization of switched linear systems with polytopic uncertainties by employing the methods of nonsmooth analysis and the composite quadratic Lyapunov functions. Above all, the minimum quadratic functions and the directional derivatives along the vertex directions of subsystems are applied to construct the new switching law. Then, some sufficient conditions for stabilization of switched linear systems are established considering the sliding modes and the directional derivatives along sliding modes. Finally, numerical examples are given to demonstrate the effectiveness of the synthesis results.

1. Introduction

It is well known that many practical systems are inherently multimodal, and several dynamical subsystems are required to describe their behavior which may depend on various environmental factors. In addition, there are some systems that sometimes cannot be asymptotically stabilized by a single continuous feedback control law but can be stabilized by switching law [1]. In the last two decades, considerable interest in the work on switched systems has attracted researchers' attention in particular and many significant results on stability and stabilization problems for various types of switched systems have been established [2–11].

As a hybrid dynamical system which is composed by a family of continuous-time or discrete-time subsystems with a rule orchestrating the switching between the subsystems [12,13], switched systems have many applications in mechanical control systems, the automatic industry, aircraft and air traffic control switching, power converters, etc.

From the practical viewpoint, it is important to investigate switched systems with uncertain parameters. As pointed out in [14], polytopic uncertainties exist in many real systems, and most of the uncertain control systems can be approximated by systems with polytopic uncertainties. The polytopic uncertain systems are less conservative than systems with norm bounded uncertainties [15]. Recently, the stability and stabilization problems for both continuous-time and discrete-time switched linear systems with polytopic uncertainties are investigated in [16,17]. In particular, [16] investigated the quadratic stabilization problem via state feedback, and provided sufficient conditions such that the switched system composed of two subsystems is quadratically stabilized. More recently, necessary and sufficient conditions for continuous-time case via state feedback are proved in [18,19].

On the other hand, increasing nonquadratic Lyapunov functions have been used in stability analysis or stabilization of the switched systems [7,8,20–23]. Considering the discrete-time systems, in [7], the authors studied the exponential

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stabilization problem for the switched linear systems based on a special control-Lyapunov function which can make the hybrid-control policy of the related switched system be derived analytically and computed efficiently. The generating functions of the switched linear systems are proposed and the exponential growth rates of the system trajectories in [8] are obtained under various switching laws. In particular, necessary and sufficient conditions for the exponential stability of the switched linear systems are derived based on these radii of convergence.

For the case of continuous-time dynamical systems, Hu and Lin [24] proposed the composed quadratic Lyapunov functions for constrained control systems. The composite quadratic Lyapunov function turned out to be very effective in dealing with some constrained control systems as well as a class of more general nonlinear systems [2,25,26].

In this paper, we focus on the continuous-time switched linear systems. The stabilization problems of switched linear systems with polytopic uncertain subsystems are studied by using composite quadratic functions. To the best of the author's knowledge, no results are reported in the literature on the stability and stabilization of switched systems with polytopic uncertainties by employing composite quadratic Lyapunov functions. The contributions in our work mainly include three aspects. First, the switching law is constructed based on the directional derivatives along the vertex directions of all subsystems. Second, some analysis results on possible sliding modes in the systems are established in this paper. Third, it is different from the approaches in Refs. [16,18,19] that the nonquadratic Lyapunov function is employed to reduce the conservatism and establish the matrix conditions for stabilization. Numerical examples show that our test results are superior to the ones in [16,18,19].

The remainder of this paper is organized as follows. In Section 2, we briefly review some preliminaries especially three composite quadratic functions and their directional derivatives. Then, the stabilization results based on the min function are established in Section 3, and the conditions as matrix inequalities for stabilization are derived. Three simulation examples are presented in Section 4 to demonstrate the effectiveness of the min quadratic Lyapunov function and the stabilization method. Finally, concluding remarks are given in Section 5.

Notation:

The following notation will be used in this paper.

- $I[k_1, k_2]$: the set of integers $\{k_1, k_1+1, k_1+2, \ldots, k_2\}$;
- $\nabla V(x)$: gradient of V at x;
- $\partial V(x)$: subdifferential of V at x;
- $\dot{V}(x;\zeta)$: one-sided directional derivative of V at x along ζ;
- $\mathrm{co}\{S\}$: convex hull of a set S.

2. Preliminaries

There are many methods to construct the switching law for the stabilization of switched systems [2,16,27]. Just as in the paper [2], our switching law is constructed by employing directional derivatives and nonquadratic functions. Now we first briefly review some preliminaries.

2.1. The directional derivative and subgradient

Suppose f is defined from R^n to $\bar{R} = R\bigcup\{-\infty, +\infty\}$, and $f(x)$ is finite. It is well known that the one-sided directional derivative of f at x in direction ζ can be expressed by

$$\dot{f}(x;\zeta) = \lim_{\Delta t\downarrow 0} \frac{f(x+\zeta\Delta t) - f(x)}{\Delta t}.$$

Suppose f is a convex function in R^n, and finite at x. The set

$$\partial f(x) = \{x^* \mid f(z) \geq f(x) + x^{*T}(z-x), \forall z \in R^n\}$$

is called the subdifferential of f at x and x^* is a subgradient of f at x [28]. For a convex function f in R^n, f is differential at x_0 if and only if $\partial f(x_0)$ has only one vector. In this case we have $\partial f(x_0) = \nabla f(x_0)$.

For a non-convex function, the subdifferential fails to exist in the convex sense. Suppose that a function f is locally Lipschitz on $\Omega \subset R^n$. The generalized directional derivative of f at x in ζ, denoted $f^0(x;\zeta)$, is defined as

$$f^0(x;\zeta) = \limsup_{\substack{y\to x\\ t\downarrow 0}} \frac{f(y+t\zeta) - f(y)}{t}.$$

The generalized gradient or the Clarke's differential of f at x, denoted as $\partial_C f(x)$, is given by

$$\partial_C f(x) = \{x^* \in R^n \mid x^{*T}\zeta \leq f^0(x;\zeta), \forall \zeta \in \Omega \subset R^n\}.$$

If f is locally Lipschitz near x and S is any set of Lebesgue measure 0 in R^n and the set of points at which f fails to be differentiable is denoted by Ω_f. Then

$$\partial_C f(x) = \mathrm{co}\left\{\lim_{x_i\to x} \nabla f(x_i) \mid x_i \notin S, x_i \notin \Omega_f\right\}. \tag{1}$$