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Equivalence of several characteristics of dual switched linear systems[☆]

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ABSTRACT

This paper is concerned with the equivalence of several dynamic characteristics between mutually dual switched linear systems, both discrete- and continuous-time cases are considered. Two systems are mutually dual means that their system matrices are transpose of each other. The dynamic properties considered in this paper include four types: exponential stability, global uniform asymptotic stability, attractivity, and weak attractivity. It is shown that every one of the four dynamic properties of a switched linear system implies each of the four dynamic properties of its dual system, and vice versa. The main results enable us to investigate these dynamic properties of switched system by virtue of study related to the corresponding properties of its dual system, thus providing an alternative way to explore the dynamics of switched systems. A numerical example is provided to illustrate the obtained theoretical conclusions.

1. Introduction

As an important class of hybrid dynamic systems, switched systems inherit the feature of both continuous state and discrete state dynamic systems. Loosely speaking, a switched system consists of a family of dynamical subsystems and a rule, called a switching signal, that determines the switching manner between the subsystems. Many dynamic systems can be modeled as switched systems [1] which possess rich dynamics due to the multiple subsystems and various possible switching signals [2–4]. Though a great deal of attention has been paid to study switched systems [5–11], there are still many unsolved and challenging issues [12]. Indeed, even for the simplest switched systems consisting of finite linear time-invariant subsystems, their stability properties are concerns of a large number of researchers [13,14].

Generally speaking, when we consider the stability property of a dynamic system, it is sometimes more convenient to establish the results by studying another system related to the original system. One of such approaches is using the equivalence of stability between two systems under duality. Two systems are said to be mutually dual if their system matrices are mutually transposed [15,16]. It is well known that a linear time-invariant system and its dual system have the same stability properties, that is, the system $\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t)$ is (asymptotically) stable if and only if the system $\dot{\boldsymbol{x}}(t) = A^{\mathrm{T}}\boldsymbol{x}(t)$. Based on this simple property, Ait Rami and Tadeo [15] proposed a new stability condition for positive systems [15, Theorem 2.4]. It is this simple result that motivates some new ideas, with which many important and nice properties of positive

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systems were revealed [17–19]. As a matter of fact, duality is widely used in a variety of fields, such as circuits, systems, and control theory [20,21].

A question naturally arises: for a switched system, does certain stability property of a system implies that of its dual system? If the answer is affirmative, then when checking the stability of a given switched system, we can fulfil the task by means of its dual system. This topic is significant, since there are many situations where we can only check the stability of the duality of a system by using the reported results, and cannot check the stability of the system itself [22]. What is more important is that with such a property holds for delay-free switched systems, it is hopeful to extend it to the case of switched systems with delays. To the best of our knowledge, it seems that no relevant answer has been reported yet. As the first step, this paper will focus on the relationships of two mutually dual delay-free switched linear systems.

Recently, Angeli et al. [23] proved that exponential stability, global uniform asymptotic stability, attractivity, and weak attractivity of a switched system of homogeneous degree one are equivalent to each other under arbitrary switching. Thus, a more interesting question also arises: Is it possible to investigate a certain dynamic property of continuous-time switched system by virtue of another dynamic property of its dual system?

Motivated by the questions mentioned above, the present paper tries to establish an equivalence relationship between two mutually dual switched systems for exponential stability, global uniform asymptotic stability, attractivity, and weak attractivity, both discrete- and continuous-time cases are treated. The main contribution of the paper lies in the following three aspects: First, as a parallel result of Theorem 2 in [23] for continuous-time switched systems, we show that these four dynamic properties of a discrete-time switched system of homogeneous degree one are equivalent to each other. Second, as far as discrete-time switched linear systems are concerned, we prove that each of the four dynamic properties can be inferred from any of the four dynamic properties of its dual system. Third, a similar result is also established for continuous-time switched linear systems. As a whole, our results provide an alternative approach to investigate the stability of switched systems.

The rest of this paper is organized as follows. Section 2 reveals the equivalence relationship of several dynamics between a discrete-time switched system and its dual system, and Section 3 deals with the same topic for continuous-time switched systems. An example is given in Section 4, and Section 5 concludes this paper.

Notations: A^{T} is transpose of matrix A. $\mathbb{R}(\mathbb{R}_{0,+}, \mathbb{R}_+$, resp.) denotes the set of real (nonnegative, positive, resp.) numbers. $\mathbb{R}^{n\times m}$ is the set of real matrices of $n \times m$-dimension and $\mathbb{R}^n = \mathbb{R}^{n\times 1}$. $\mathbb{N}_0$ is the set of nonnegative integers and $\mathbb{N} = \mathbb{N}_0 \setminus \{0\}$. For any $m \in \mathbb{N}$, define $\underline{m} = \{1, 2, \ldots, m\}$ and $\underline{m}_0 = \underline{m} \cup \{0\}$. $|a|$ is the absolute value of a real number a and $\|\boldsymbol{x}\|_\infty$ the l_∞ norm of vector $\boldsymbol{x}$. Define $\mathcal{B}_\varepsilon = \{\boldsymbol{x} \in \mathbb{R}^n | \, \|\boldsymbol{x}\|_\infty < \varepsilon\}$, $\bar{\mathcal{B}}_\varepsilon = \{\boldsymbol{x} \in \mathbb{R}^n | \, \|\boldsymbol{x}\|_\infty \le \varepsilon\}$, $\mathcal{B}(C, \varepsilon) = \left\{\boldsymbol{x} \in \mathbb{R}^n | \inf_{\boldsymbol{\xi}\in C} \|\boldsymbol{x} - \boldsymbol{\xi}\|_\infty < \varepsilon,\ C \subset \mathbb{R}^n\right\}$. For a set $C \subset \mathbb{R}^n$, $\|C\| = \sup_{\boldsymbol{\xi}\in C} \|\boldsymbol{\xi}\|_\infty$ and $\lambda C = \{\lambda\boldsymbol{\xi} | \boldsymbol{\xi} \in C\}$ with $\lambda \in \mathbb{R}^+$. Given matrix $A \in \mathbb{R}^{n\times n}$, let $\mathbf{M}(A) = \max_{1\le i,j\le n} \mid a_{ij} \mid$ with a_{ij} being the (i,j)th element of A, $(A)_j$ be the jth column of matrix A, $\|A\|_1$ and $\|A\|_\infty$ the ℓ_1 and ℓ_∞ matrix norms, respectively, that is, $\|A\|_1$ is the maximum absolute column sum of A and $\|A\|_\infty$ the maximum absolute row sum of A [24]. Throughout this paper, the dimensions of matrices and vectors will not be explicitly mentioned if clear from context.

2. Discrete-time switched systems

In this section, we will first explore an interesting property of a discrete-time control system. Then, we will establish the equivalence relationships between four dynamic properties (exponential stability, global uniform asymptotic stability, attractivity, and weak attractivity) of switched homogeneous systems. Finally, we show that two dual switched linear systems have the same dynamic properties.

2.1. Finite first crossing time of discrete-time control systems

Consider the following control system:

$$\boldsymbol{x}(k+1) = \boldsymbol{f}\left(\boldsymbol{x}(k), \boldsymbol{u}(k)\right), \quad k \in \mathbb{N}_0, \tag{2.1}$$

where $\boldsymbol{x}(k) \in \mathbb{R}^n$ is the state variable, $\boldsymbol{u}(k)$ is the control input. In the present paper, we assume that $\boldsymbol{u}(k)$ takes values in a compact set $\mathcal{U} \subseteq \mathcal{X}$ with $(\mathcal{X}, d)$ being a metric space (the set $\mathcal{X}$ equipped with a metric d). Denote $\mathcal{M}_\mathcal{U}$ the set of (infinite) sequences of elements of $\mathcal{U}$. We make the following assumption for (2.1):

Assumption 1. $\boldsymbol{f} : \mathbb{R}^n \times \mathcal{U} \to \mathbb{R}^n$ is continuous in the second argument and Lipschitz in the first argument uniformly with respect to the second one. That is, there exists a positive scalar L such that

$$\|\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) - \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{u})\|_\infty \le L\|\boldsymbol{x} - \boldsymbol{y}\|_\infty, \quad \forall \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n, \forall \boldsymbol{u} \in \mathcal{U}.$$

Note that if $\boldsymbol{f}$ is continuous in $\boldsymbol{x}$ and $\boldsymbol{u}$ separately, then it is not necessarily continuous on $\mathbb{R}^n \times \mathcal{U}$. However, if $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$ satisfies Assumption 1, then $\boldsymbol{f}$ is necessarily continuous on $\mathbb{R}^n \times \mathcal{U}$. To show this fact, arbitrarily pick $(\boldsymbol{x}_0, \boldsymbol{u}_0) \in \mathbb{R}^n \times \mathcal{U}$. Since $\boldsymbol{f}$ is continuous in the second argument, for fixed $\varepsilon > 0$, there exists a scalar $\delta_1(\boldsymbol{x}_0, \boldsymbol{u}_0) > 0$ such that $\|\boldsymbol{f}(\boldsymbol{x}_0, \boldsymbol{u}) - \boldsymbol{f}(\boldsymbol{x}_0, \boldsymbol{u}_0)\|_\infty < \frac{\varepsilon}{2}$ if $d(\boldsymbol{u}, \boldsymbol{u}_0) < \delta_1(\boldsymbol{x}_0, \boldsymbol{u}_0)$. Clearly, $\|\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) - \boldsymbol{f}(\boldsymbol{x}_0, \boldsymbol{u})\|_\infty \le L\|\boldsymbol{x} - \boldsymbol{x}_0\|_\infty$ for all $\boldsymbol{x} \in \mathbb{R}^n$ and