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Global stabilization design for switched power integrator triangular systems with different powers

Ruicheng Ma [a,*], Yan Liu [b], Shengzhi Zhao [a], Min Wang [c], Guangdeng Zong [d]

[a] School of Mathematics, Liaoning University, Shenyang, 110036, China
[b] College of Information Science and Engineering, Northeastern University, Shenyang, 110819, China
[c] Department of Mathematics, China University of Mining and Technology, Xuzhou, 221008, China
[d] Institute of Automation, Qufu Normal University, Qufu, 273165, China

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ABSTRACT

In this paper, we will study the global exponential stabilization (GES) under arbitrary switchings for a class of switched nonlinear systems in power integrator triangular form, whose subsystems have chained integrators with the powers of positive odd numbers. Unlike the existing results on systems where the powers are assumed to be identical in $\dot{x}_i$-equation, all the powers in each equation of subsystems of the switched systems can be different. Based on the unbounded time-varying scaling of the states, both a class of state-feedback controllers of individual subsystems and a common Lyapunov function (CLF) are explicitly constructed by a recursive design algorithm to guarantee global exponential stability of the closed-loop switched system under arbitrary switchings. In the controller design, a common transformation of all subsystems is exploited to avoid using individual coordinate transformation for each subsystem, which is achieved by establishing the relationship of the powers. Finally, a numerical example is given to illustrate the effectiveness of the proposed results.

1. Introduction

Switched systems as an important class of hybrid systems have drawn considerable attention in recent years. Switched systems arise in engineering practice where system behavior can hardly be described by a single ordinary differential equation and several dynamical system models are required to model an engineering system due to the various jumping parameters and changing environmental factors, such as, gene regulatory networks, multi-agent systems, robotic, switching power converters, and automotive industry [1–5]. The main concerns in study of switched systems are the issues of stability and stabilization (see, for example, [6–15]). Many significant methods have been proposed to study these problems, such as a CLF, multiple Lyapunov functions, and switched Lyapunov functions [15–27]. Stability under arbitrary switchings is a desirable property of switched systems due to its practical importance, and is also a fundamental problem in the analysis and design of switched systems. For this problem, it is necessary that all the subsystems be asymptotically stable. However, it is possible to have unstable trajectories when switching among stable systems [16]. It is well known that if there exists a common Lyapunov function for all the subsystems, then the stability of the switched system is guaranteed under arbitrary switchings. The problem on the existence of a CLF has been studied and some important results have been obtained [16,28]. For the switched linear systems, a quadratic CLF is constructed in [16,28]. When the uncertain nonlinear functions are constrained in a sector set, [29] proposed a sufficient condition to guarantee the existence of a CLF for the switched systems. [30]

* Corresponding author. Tel.: +86 13840184679.
E-mail address: mrccheng@163.com (R. Ma).

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provided a way to search for a CLF for switched nonlinear systems of polynomial form. However, it is worth pointing out that, since there is no systematic approach to construct a CLF for general nonlinear systems, we encounter the same difficulty to construct such a CLF for general switched nonlinear systems. Thus, attention is shifted to switched nonlinear systems with special structures as system structures may play an important role in constructing such a CLF.

On the other hand, triangular structure is a popular system structure for switched nonlinear systems, which is not only theoretically interesting but also important from the application point of view, such as, in application to a continuously stirred tank reactor system (CSTR) [31–33]. Strict-feedback systems are an important class of lower triangular systems. For a class of switched strict-feedback systems with switching jumps, adaptive neural control has been presented in [34]. [31,35] obtained the global stabilization under arbitrary switchings for switched strict-feedback systems. The global robust stabilization result has been obtained in [7] for switched strict-feedback systems with zero dynamics. Another important class of triangular system is the power integrator triangular system, which includes the strict-feedback system as a special case. The power integrator triangular system has been studied in [36–39] and has many applications, such as, in the underactuated unstable mechanical system [37]. [40] considered a class of switched nonlinear systems in p-normal form. One feature of the studied switched systems in the mentioned references above is that the powers of the chained integrators are restricted to the same positive odd integer for subsystems, and thus these methods are not applicable to switched systems where all powers in each equation of subsystems of the switched systems can be different. [32,33] studied the more general switched systems in p-normal form, where the powers of the subsystems are allowed to be different and to be positive even integers, and obtained sufficient conditions under which a globally asymptotic stabilizer is designed. Questions naturally arise: *under arbitrary switchings, is it possible to exponentially or asymptotically stabilize smooth switched nonlinear systems whose powers of integrators are different for different periods of the whole operation time? If possible, under what conditions can such controllers be designed and how?* To our best knowledge, in the literature there have not been results to answer these questions. The key to solve these problem is how to simultaneously construct state feedback stabilizers and a CLF by the construction design method. Since the powers in each equation of subsystems of the switched system are positive odd numbers and can be different, the switched system has an uncontrollable Jacobian linearization at the origin, and also makes the standard adding a power integrator technique not applicable to construct a CLF. In addition, the hybrid features of switched systems make stabilization more difficult than non-switched systems, which impose another challenge.

To address these challenging problems, in this paper we will focus on the case where the powers of the chained integrators (of switched nonlinear systems) are positive odd numbers and can be different for all subsystems. Note that no results even on the *asymptotic* stabilization under arbitrary switchings are available in the literature. It is worth pointing out that even if the switched systems was somehow stabilized during its corresponding period by the method or its extension of [36,39], the closed-loop switched system can be still unstable due to the fact that it is possible to get instability of switched systems by switching between stable subsystems [16]. Therefore, we will present the stabilizer design and study the global exponential stabilization of switched power integrator triangular systems, where the chained integrators have the powers of positive odd numbers, and the powers of the chained integrators can be different for each subsystem during the whole operation time. Specifically, when the power of the chained integrators is different, the traditional methods are in general no longer applicable. This is because the virtual stabilizing functions for each subsystem are often different due to the different powers, which lead to different coordinate transformations for different subsystems. This means that we have to construct a CLF under different coordinate transformations which is extremely difficult. Therefore, to obtain a common coordinate transformation for all subsystems during iterations is desired, which is also a crucial issue to employ the constructive technique to switched nonlinear systems. However, to the best of authors' knowledge, this issue has not been addressed in the existing literature, which also partly motivates our present work.

To achieve the control objective of this paper, the unbounded time-varying scaling of the states for global exponential stabilization of studied switched systems is firstly derived, which brings the switched system into a specific form. A recursive design algorithm is developed to construct a class of state-feedback stabilizer as well as a C^1 positive definite and proper common Lyapunov function, which guarantee that the scaled states exponentially converge to the origin. Combining the properties of the scaling with the convergence of the scaled states, we deduce GES of the state and estimate the rate of convergence, while the control signal is also bounded and converges to zero. Compared to the relevant existing results in the literature, the main contributions of this paper are as follows. In this paper the global *exponential* stabilization under arbitrary switchings has been obtained by the constructive design method for a class of switched nonlinear systems in power integrator triangular form for the first time. On the other hand, the switched system we considered in this paper includes the switched strict-feedback systems considered in [31,35] as a special case where achieved only global *asymptotic* stabilization. Obviously, global exponential stabilization is more desired than global asymptotic stabilization. This paper, for the first time, also applies the unbounded time-varying scaling of the states method to switched nonlinear systems in power integrator triangular form, and overcomes the obstacle imposed by the different powers of the chained integrators of each equation. With the unbounded time-varying scaling of the states method, we mathematically and recursively develop an algorithm to simultaneously construct a global exponential stabilizer and a common Lyapunov function for the switched nonlinear system considered. Of course, the proposed method also applies to the corresponding non-switched nonlinear system (i.e., the switched system has only one subsystem).

This paper is organized as follows. Section 2 introduces preliminaries and system description. Section 3 gives the main result in this paper. An example is given to show the effectiveness of the proposed scheme in Section 4. Finally, some conclusions are drawn in Section 5.