Nonlinear Analysis: Hybrid Systems 15 (2015) 86–97

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Stochastic piecewise affine control with application to pitch control of helicopter

Kaveh Merat, Hassan Salarieh *, Aria Alasty, Ali Meghdari

Department of Mechanical Engineering, Sharif University of Technology, P.O. Box 11155-9567, Azadi Ave, Tehran, Iran

ARTICLE INFO

Article history:
Received 16 November 2013
Accepted 4 August 2014

Keywords:
Additive noise
Piecewise affine systems
Ito differential
Stochastic hybrid control
Helicopter pitch control

ABSTRACT

In this paper, at first the stability condition which gives an upper stochastic bound for a class of Stochastic Hybrid Systems (SHS) with deterministic jumps is derived. Here, additive noise signals are considered that do not vanish at equilibrium points. The presented theorem gives an upper bound for the second stochastic moment or variance of the system trajectories. Then, the linear case of SHS is investigated to show the application of the theorem. For the linear case of such stochastic hybrid systems, the stability criterion is obtained in terms of Linear Matrix Inequality (LMI) and an upper bound on state covariance is obtained for them. Then utilizing the stability theorem, an output feedback controller design procedure is proposed which requires the Bilinear Matrix Inequalities (BMI) to be solved. Next, the pitch dynamics of a helicopter is approximated with a set of linear stochastic systems, and the proposed controller is designed for the approximated model and implemented on the main nonlinear system to demonstrate the effectiveness of the proposed theorem and the control design method.

1. Introduction

The stability and control of stochastic hybrid systems (SHS) have attracted many research works over the past decade. Related to hybrid systems, the stability of deterministic systems was among the first studied subjects. One of the most applied and well known methods is the multiple Lyapunov function method [1,2]. Specifically, stability and control of switching piecewise affine systems which are based on the multiple Lyapunov methods were studied by many authors as in [3–5]. As for the deterministic controller design procedures utilizing the multiple Lyapunov function, Hassibi and Boyd [6], Rantzer and Johansson [7], and Rodrigues and Boyd [5] considered deterministic systems controlled by continuous-time controllers.

The stochastic hybrid systems are an extension of hybrid systems by adding randomness. This randomness could be added to the jumps or subsystems or both. One of the first frameworks for stochastic hybrid systems which includes both randomness in jumps and diffusion in continuous-time systems is introduced in [8]. In [8], the rest-maps of jumps are stochastic but still the jump conditions are considered to be deterministic. The Generalized Stochastic Hybrid Systems (GSHS) with probability kernels for jumps are given in [9], where a formula for the extended generator of such systems is also derived.

On the other hand, the stability of stochastic systems based on the extension of the Lyapunov second method has been widely investigated. Among the pioneers, Kushner in his book [10] gathered many theorems regarding the stochastic stability which are mainly based on Dynkin's formula. More recently, in [11] Lyapunov-like theorems addressing qualitative

* Corresponding author. Tel.: +98 21 6616 5538; fax: +98 21 6600 0021.
E-mail addresses: salarieh@sharif.ir, salarieh@sharif.edu, hsalarieh@yahoo.com (H. Salarieh).

http://dx.doi.org/10.1016/j.nahs.2014.08.001
1751-570X/

properties such as stochastic boundedness, stochastic stability, stability of moments for Ito stochastic differential equations are reviewed.

Deriving stability criteria for different classes of stochastic hybrid systems has also been studied more recently. Abate, et al. [12] give the asymptotic stability condition for SHS with Markov process as jumps which have Probabilistic dwell-time. In [13], the stochastic Lyapunov stability theorems for SHS with arbitrary switching are considered where noise terms vanish at the origin, so that the stability of origin with probability one could be proved. The almost sure exponential stability conditions for a class of nonlinear switching Ito stochastic systems are studied in [14], again requiring the noise terms to become zero at the origin. Similarly, in [15] a multiple Lyapunov theorem is proposed for the exponential m-stability analysis of stochastic switched systems with complete arbitrary jumps. The LaSalle stability theorem is extended to general stochastic hybrid systems in [16] which again guarantees exponential stochastic stability. In [17], a stability theorem regarding stopping times for SHS with state-dependent jumps is proposed. But in the main theorem of the mentioned article which is an extension of the output feedback control design of the Barbalat Lemma, again vanishing noise at the equilibrium point is assumed.

As for stochastic feedback controllers for hybrid systems, the problem of hybrid control for a class of stochastic nonlinear Markovian switching systems under impulsive control is addressed in [18]. In [19], the sliding mode control of nonlinear singular stochastic systems with Markovian switching is investigated. Boukas investigated the output feedback control problem of an uncertain singular linear stochastic system with Markov switches in [20], where the uncertainties are norm bounded and the stochastic noise is vanishing at the origin. The output feedback control design of a continuous-time linear system with Markovian jump is also studied in [21], which leads to an LMI formulation to provide the mean-square stability for the whole system which indicates that the variance of states tends to zero as time advances. Moreover in [22], for linear cases, via the LMI method, the controller design methods are proposed using almost surely exponentially stability theorem.

Most of the papers in the literature of control and stability analysis of stochastic hybrid systems have not considered state-dependent deterministic jumps and additive noise. Both of the aforementioned properties are important for engineering models as considered here. As mentioned before, in a multiplicative noise with vanishing noise amplitude at the equilibrium point, asymptotic or exponential stochastic stability could be obtained. But for the models with the additive noise considered here, only a bound on the variance of states could be achieved and non-vanishing noise in the origin prohibits the application of exponential stochastic stability theorems. As for the state-dependent deterministic jump property, the whole state space is partitioned into some regions where in each one, a different Ito deferential equation governs.

One of the most relevant works to the present work is [23]. In [23], a piecewise stochastic system with additive noise is studied, but the common Lyapunov function method is applied and it is assumed that drift terms and noise terms are continuous at boundaries between subsystems. Extending the previous work, in the present work, the multiple Lyapunov function is applied and the constraint on the continuity of drift and noise terms on boundaries is lifted and a bound for variance of them is also found. So, the main contribution of this paper is the stability theorem which gives an upper bound on the stochastic second moment for a class of SHS with non-vanishing noise in the origin which can be viewed as a piecewise stochastic system. Then for a more special case that consists of linear stochastic systems in polyhedral regions, the stability conditions in an LMI format are derived. In the controller design procedure, due to the introduction of controller gains, BMI conditions arise. So, the convex–concave decomposition optimization introduced in [24] is applied to develop a systematic method for the controller design. It is also notable that as the systematic control design procedures for piecewise affine stochastic systems are derived, it could be exploited for the controller design in nonlinear stochastic systems using their piecewise linear stochastic approximation.

This paper is organized into six parts. In Section 2, the problem formulation and assumptions are introduced. In Section 3, the stochastic stability theorem is proved for the piecewise affine system based on the more general theory. Then a method to design the stochastic feedback controller for such systems is developed in Section 4. Afterward in Section 5, a nonlinear model of helicopter pitch dynamics and its piecewise approximation are given and the proposed controller design is implemented and simulation results are presented to show the performance of the method. Finally, Section 6 is the conclusion for this paper.

2. Piecewise stochastic system and preliminaries

A general family of piecewise nonlinear stochastic hybrid systems in Ito form with state-dependent jumps could be defined as

$$dx = f_i(x)\,dt + g_i(x)\,d\omega, \quad x \in \Omega_i,\ i \in I \tag{1}$$

where $\Omega_i \subseteq \Re^n$ denotes a partition of the state space with I as the index set of regions where $\cup_{i\in I}\Omega_i = \Re^n$. The variable $x(t) \in \Re^n$ is the state vector, $\omega(t)$ is a q-dimensional normalized Wiener process satisfying $E[d\omega] = \mathbf{0}$ and $E\left[(d\omega)(d\omega)^T\right] = J_{q\times q}dt$ with $J_{q\times q}$ as the q-dimensional identity matrix. dx is the stochastic differential of x and $f_i(x) : \Re^n \to \Re^n$ and $g_i(x) : \Re^n \to \Re^{n\times q}$ for $\forall i \in I$ are bounded and Lipschitz continuous to ensure the existence and uniqueness of the corresponding solution process in Ω_i.

In this paper, mostly the following piecewise affine stochastic system which is a special form of (1) is considered:

$$dx = (A_i x(t) + a_i)\,dt + S_i d\omega, \quad x \in \Omega_i,\ i \in I. \tag{2}$$