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Geometric properties of solution of a cylindrical dynamic system with impulsive state feedback control

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ABSTRACT

A kind of cylindrical dynamic system with impulsive state feedback control is formulated and investigated. Based on the qualitative properties of the corresponding continuous system, the existence of order-k ($k \in \mathbf{Z}^+$) periodic solutions of the cylindrical dynamic system with impulsive state feedback control is discussed on the cylinder with perimeter 2π. If the equilibrium of the corresponding continuous system in the rectangle coordinate system is an unstable node, then the cylindrical dynamic system has two one-side stable minimum limit sets. If the equilibrium is an unstable focus, then, for different parameters, the cylindrical dynamic system has the periodic solutions with different periods and different orders. Finally, numerical simulations are given to verify the theoretical results.

1. Introduction

In the development and management process of biological species, the implementations of some control measures depend on the state of target species, that is, only when the population or density of the target species reaches a certain threshold (e.g., economic threshold, ET), the corresponding control measures are implemented. Otherwise, no control measure is taken. This kind of control is known as state feedback control. Because those control measures have the characteristics of pulse-like actions, they are also called impulsive state feedback control. Since the impulsive state feedback control can be widely used in many biological systems and the differential equations with impulsive state feedback control can provide a natural description of the pulse-like actions, the differential equation with impulsive state feedback control receives more and more attention of researchers engaged in the study of biomathematics and other fields.

The autonomous differential equation with the term of impulsive state feedback control was called impulsive semi-dynamic system in Ref. [1] and some abstract properties (e.g., limit set) were given there. In the past years, the application of impulsive semi-dynamic system in the fields of biomathematics mainly focused on the models of pest control, microbial cultivation and disease control, the geometric properties of solution of these impulsive semi-dynamical systems were investigated clearly. For example, Refs. [2–5] studied the state-dependent impulsive systems of integrated pest management (IPM) strategies and gave the corresponding dynamic consequences. By using the method of bifurcation, Refs. [6,7] studied the impulsive state feedback control of prey–predator system and gave the existence and stability of order-1 periodic solution. Whereafter, various of prey–predator systems with impulsive state feedback control were investigated.

In the control process of microorganism culture, a turbidostat is an apparatus with feedback control system used to continuously culturing microorganisms. The dilution rate of the turbidostat can be regulated by the control system when the concentration of microorganism, detected by photoelectricity system or other devices, reaches a preset value. Based

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on the design ideas of the turbidostat, the differential equation with impulsive state feedback control was proposed in Refs. [8–10] and investigated by the existence criteria of periodic solution of a general planar impulsive autonomous system which generalized the Poincaré–Bendixson theorem [11], the conditions for the existence of order-1 periodic solution were obtained according to the preset value and the types of positive equilibrium of the corresponding continuous system. For different kinds of microorganism cultures, various differential equations with impulsive state feedback control were formulated and investigated, see Refs. [12–15].

For the disease control, Ref. [16] proposed two mathematical models with impulsive injection of insulin or its analogues for type 1 and type 2 diabetes mellitus. One model incorporated the periodic impulsive injection of insulin. The other model determined the insulin injection by closely monitoring the glucose level. The existence and stability of order-1 periodic solution were proved to explain that the perturbation by the injection in such an automated way can keep the blood glucose concentration under control.

With the application and further research of impulsive semi-dynamical system in the fields of biomathematics, some new geometric properties were found, then Ref. [17] summarized the characteristics of the biological dynamic systems with impulsive state feedback control and called it semi-continuous dynamic system. The definitions and preliminary research methods of semi-continuous dynamic system were given in Ref. [17]. Most of early researches on the biological system with impulsive state feedback control considered that the implementation of control measure only depends on one target species and the function of impulsive condition only involves one variable (see Refs. [18,19] and the papers mentioned above). But in some habitats where the resources (e.g., food, space) are limited, when the total population of the species in the habitat reaches a certain threshold, the resources will become scarce and cannot meet the need of the species to survive. At this time, some control measures, aiming at all the species in the habitat, not a single special species, should be taken to maintain the growth of all the species. To describe this kind of control condition, the function of impulsive condition will involve two or more variables.

The references mentioned above consider that the impulsive conditions are linear functions with one variable. From the geometry point of view, most of them are either the horizontal straight lines or the vertical lines in the plane. But there is few paper to discuss the case in which the function of impulsive condition is quadratic. This paper will formulate and discuss a kind of linear species system in which the function of impulsive condition is quadratic and its geometric curve is a circle, which can be considered as the reference to discuss the nonlinear biological systems in which the functions of impulsive conditions are also quadratic.

On the other hand, the differential systems given in the references mentioned above do not involve the cylindrical dynamic system. To use the existing knowledge of linear impulsive condition, this paper will formulate a kind of linear system and transform it into a cylindrical system with perimeter 2π by polar transformation, and then mainly investigate the geometric properties of solution of cylindrical system with impulsive state feedback control.

The rest of this paper is organized as follows. In Section 2, we will introduce a continuous system which can be viewed as a predator–prey system, and its semi-continuous system with impulsive state feedback control in the rectangle coordinate system. By polar transformation, the semi-continuous cylindrical system with impulsive state feedback control is formulated. The qualitative properties of the corresponding continuous system are given in Section 3. Section 4 will show that the semi-continuous cylindrical system has order-k ($k \in \mathbf{Z}^+$) periodic solutions with different periods and different orders as the parameter changes. The orbit stability of periodic solution is discussed in Section 5. Numerical simulations and discussions are given in Section 6.

2. Model formulation

Suppose that there are two species in a habitat where the food resource is limited. Denote the populations of the species by $x(t)$ and $y(t)$ at time t, respectively. For simplicity, let $x = x(t)$ and $y = y(t)$. Suppose that the species y has the negative effect on the species x and decreases the growth rate of the species x, but the species x can increase the growth rate of the species y. The relations and evolution process of two species can be described by the following system:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = -y + \delta x, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = x, \end{cases} \tag{2.1}$$

where $\delta \geq 0$ represents the immigration rate of species x from the outside of the habitat. System (2.1) can be viewed as a simple prey–predator system. It can be easily obtained that equilibrium $O(0, 0)$ of system (2.1) is a center for $\delta = 0$, an unstable focus for $0 < \delta < 2$ and an unstable node for $\delta \geq 2$.

From these results, we know that the population of two species x and y tend to infinite, that is, $x \to +\infty$ and $y \to +\infty$ as $t \to \infty$ for $\delta > 0$. But the food is limited, the populations of two species will decrease even tend to zero after the total population reaches a certain threshold because of the lack of food. At this time, some control measures should be taken. We suppose that the threshold satisfies $\sqrt{x^2 + y^2} = r_1,\ 0 < r_1$ and the initial state is $\sqrt{x_0^2 + y_0^2} < r_1$ where x_0 and y_0 are the initial values of x and y at the initial moment $t = t_0$. As the time t increases and the point moves along the trajectory of system (2.1), if x and y satisfy $\sqrt{x^2 + y^2} = r_1$, then some control measures can be taken and suppose that the control