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An implicit systems characterization of a class of impulsive linear switched control processes. Part 1: Modeling

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ABSTRACT

Our paper focuses on a fundamental structural property associated with a family of linear switched control systems in the presence of impulsive dynamics. We consider dynamic processes governed by piecewise linear ODEs with controlled location transitions and describe the resulting system in a constructive form of an implicit dynamic system. The proposed algebraic-based modeling framework follows the celebrated behavioral approach (see Polderman and Willems (1998)) and makes it possible to apply to the switched dynamics some conventional techniques from the well-established time-invariant implicit systems theory. The analytic results of our paper constitute a formal theoretical extension of switched control systems methodology and can be used (as an auxiliary step) in a concrete control design procedure. In this first part, we consider modeling aspects.

1. Introduction

This paper discusses a behavioral based analytic approach to a class of impulsive linear switched dynamic systems with controlled location transitions (see *e.g.*, [1]). The study of various types of hybrid and switched systems has gained lots of interest in the recent years (see [2–13]). Many real-world applications from different disciplines have been studied in the modeling framework that involves switched structures. The general switched systems constitute a wide class of modern control systems where two types of dynamics are present, continuous and discrete event dynamics (see e.g. [14,15]). In order to understand how these systems can be operated efficiently, both aspects of the dynamics have to be considered and taken into account during a concrete control design phase. An analytic approach that combines tools from discrete event systems and continuous systems is the most realistic but at the same time, probably mostly sophisticated. Evidently, each "component" of the complete dynamics contributes to the complexity of the resulting control design problem, namely, the combinatorial aspect of the discrete event part and the infinite dimensional problem associated with the conventional continuous dynamics. Difficulties also arise from the mutual interaction/coupling of the two above-mentioned dynamic components. For example, the stability analysis or the optimal design for switched systems are such intricate examples of the new systems phenomena (see e.g., [14,1]).

The aim of our contribution is to show that a constructive analysis of a certain class of impulsive linear switched systems with controllable location transitions can also be incorporated into the classic behavioral concept and studied using the

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basic facts from the implicit systems theory. In this connection we are concentrated on the systems modeling framework. Next we will discuss some aspects of the concrete conventional design procedures. The behavioral approach (see e.g., [16]) to the class of switched systems under consideration constitutes a theoretic extension of the modern switched systems theory. This extension can provide a basis for the corresponding computational techniques associated with a specific control design. Recall that the above mentioned techniques are widely used in the conventional linear systems theory and also can be generalized to hybrid systems and systems with switched structure [17,18]. In this paper, we first rewrite the initially given linear switched system and obtain an auxiliary dynamic model described by an equivalent implicit differential equation. The latter can be formally separated into a dynamic part (given by an ordinary differential equation) and an algebraic part. We refer to [19,20] for some standard facts from the theory of implicit differential equations and differential algebraic systems. Using the proposed algebraic approach, the initial linear system with switched structure can be finally replaced by an equivalent differential algebraic system that contains a switching-free linear differential equation and switched algebraic conditions. The resulting differential algebraic model with the obtained switching-free dynamic part admits an easier analytical treatment in comparison with the initial switched system.

The remainder of our paper is organized as follows. Section 2 specifies a class of switched systems under consideration and includes some necessary mathematical preliminaries. Section 3 contains our main theoretic result and shows how to represent a linear switched system under consideration in the form of an implicit dynamic system. Section 4 studies the fundamental structural property of the obtained implicit systems representation. In [21], we study the reachability of the proposed representation, and we propose a proportional and derivative feedback control law for making unobservable the internal variable structure, and also a proper approximation is given. Section 5 summarizes the contribution of this first part. All the proofs are sent to Appendices.

2. Linear switched systems with autonomous location transitions: the behavioral approach

In this paper, we use the fundamental dynamic system concept of Willems [22] (see also [16]), namely, the celebrated behavioral approach in analysis of a specific class of general hybrid systems with linear structure. We refer to [23] for some necessary technical details. Let us first characterize the class of systems under consideration.

Definition 1. A linear switched system (LSS) with autonomous location transitions is a eight-uple

$$\Sigma_{sw} := \{\mathcal{Q}, \mathfrak{X}, \mathbb{U}, \mathbb{Y}, \mathbf{A}, B, \mathbf{C}, \mathcal{S}\}$$

where

- $\mathcal{Q}$ is a finite set of indexes (locations);
- $\mathfrak{X} = \left\{\mathscr{X}_q\right\}$, $q \in \mathcal{Q}$, where $\mathscr{X}_q \subseteq \mathbb{R}^{\bar{n}}$, is a family of state spaces;
- $\mathbb{U}$ is a set of admissible control input functions, $u : \mathbb{R}^+ \to U, U \subseteq \mathbb{R}^m$;
- $\mathbb{Y} = \left\{C_q\mathscr{X}_q\right\}$, $q \in \mathcal{Q}$ where $C_q\mathscr{X}_q \subseteq \mathbb{R}^p$, is a family of subspaces, called the set of output values;
- $\mathbf{A} = \left\{A_q \in \mathbb{R}^{\bar{n}\times\bar{n}}\right\}$, $\mathbf{C} = \left\{C_q \in \mathbb{R}^{p\times\bar{n}}\right\}$, $q \in \mathcal{Q}$ are families of systems matrices and $B \in \mathbb{R}^{\bar{n}\times m}$;
- $\mathcal{S}$ (the switching set) is a subset of Ξ, where

$$\Xi := \left\{\left(q, q'\right) : q, q' \in \mathcal{Q}\right\}.$$

Note that Definition 1 is a specific case of the general concept from [2,5,24,7,8,10,12,13]; see also [25]. As one can see, the sub-systems switching rules (given by the set $\mathcal{S}$) are independent on the state variable. Even that switching mechanism is usually indicated as "autonomous location transition" rule. In the following, we assume that the control set U is compact and: $\mathbb{U} \subseteq \mathcal{L}_1^{\mathrm{loc}}(\mathbb{R}^+, U)$. We also assume that: $\mathbb{Y} \subseteq \mathcal{L}_1^{\mathrm{loc}}(\mathbb{R}^+, \mathbb{R}^p)$. Here, $\mathcal{L}_1^{\mathrm{loc}}(\mathbb{R}^+, \mathbb{R}^\mu)$, $\mu \in \mathbb{N}$, is the space of locally integrable functions, $\psi : \mathbb{R}^+ \to \mathbb{R}^\mu$.

The switching times associated with a LSS, $T_i, i \in \mathbb{N}$, are given sequentially as:

$$0 = T_0 < T_1 < \cdots < T_{i-1} < T_i, \dots \quad i \in \mathbb{N}, \text{ with } \lim_{i\to\infty} T_i = \infty.$$

In our paper, we assume that the switching times $\{T_i\}$ are given *a priori*. A switched control system remains in location $q_i \in \mathcal{Q}$ for all time instants $t \in [T_{i-1}, T_i)$, $i \in \mathbb{N}$. Let us also note that the pair (q, x), $q \in \mathcal{Q}, x \in \mathbb{R}^{\bar{n}}$, represents the state of a switched system at time t. Let us define: $\mathfrak{I} := \left\{\mathcal{I}_i = [T_{i-1}, T_i) \subset \mathbb{R}^+\right\}$. The restrictions of $\mathbb{U}$ and $\mathbb{Y}$ on a time interval, $\mathcal{I}_i$, is next denoted as:

$$\mathbb{U}_{\mathcal{I}_i} := \left\{u(\cdot) \in \mathcal{L}_1^{\mathrm{loc}}(\mathcal{I}_i, \mathbb{R}^m)\right\} \quad \text{and} \quad \mathbb{Y}_{\mathcal{I}_i} := \left\{y(\cdot) \in \mathcal{L}_1^{\mathrm{loc}}(\mathcal{I}_i, \mathbb{R}^p)\right\}.$$

Following the behavioral approach, we now introduce the following formal concept (called "behavior")

$$\begin{aligned}\mathfrak{B}_{sw} := \Bigg\{&(q_i, u(\cdot), y(\cdot)) \in \mathcal{Q} \times \mathbb{U} \times \mathbb{Y} \mid \exists\, c_i \in \mathbb{R}^{\bar{n}},\ \mathfrak{s} : \mathfrak{I} \to \mathcal{Q},\ \text{s.t.: } \mathfrak{s}(\mathcal{I}_i) = q_i,\\ &y(t) = \sum_{i\in\mathbb{N}} \mathbf{1}_{\mathcal{I}_i}(t) C_{q_i}\Big(\exp\left(A_{q_i}(t - T_{i-1})\right) c_i\\ &+ \int_0^{t-T_{i-1}} \exp\left(A_{q_i}(t - T_{i-1} - \tau)\right) Bu(T_{i-1} + \tau)\mathrm{d}\tau\Big),\ t \in \mathbb{R}^+,\ \mathcal{I}_i \in \mathfrak{I}\Bigg\},\end{aligned} \tag{2.1}$$