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Quantized feedback stabilization for a class of linear systems with nonlinear disturbances

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ABSTRACT

The quantized feedback stabilization problem for a class of linear system with nonlinear disturbances is addressed, in which the system and controller are connected via a communication channel. In this case, the effect of quantization errors is studied. A practical quantized scheme is designed such that the transmission error decays to zero exponentially. Meanwhile, a sufficient condition in terms of linear matrix inequality for input-to-state stability (ISS) of the system is presented with regard to the transmission error. Therefore, a quantized stabilization of the system is guaranteed based on the ISS property. A simulation example for the single-link flexible joint robot is presented to demonstrate the effectiveness of the result.

1. Introduction

In the past decades, the robust feedback stabilization of control systems has attracted much attention due to its great potential applications in control theory, mechanical systems and other research areas, and many important methods have been proposed (see [1–4], and the references therein). Robust control problems via measurement feedback for a class of nonlinear systems were investigated by a small gain control technique in [5,6]. The authors of [7] established equivalence between the existence of smooth Lyapunov functions and the robust stabilization under measurement disturbances. Ref. [8] studied the robust control design problem for a class of nonlinear systems with measurement errors based on the cyclic-small-gain theorem. In [9], the robust stabilizability with respect to sensor disturbances was addressed by the way of a counterexample; the obtained results showed that globally internal stabilizability didn't imply the globally external stabilizability for sensor disturbances.

Recently, the problem of quantized feedback stabilization on control systems is attracting more attention, where communication between the system and the controller is limited due to capacity constraints. When the control loop is closed via a digital network that transmits signals, the control system analysis and design will be more complicated than the conventional ones because a communication issue arises. As we know, any communication network can only carry limited information per unit of time. In many applications, this limitation has significant constraints on the operation of such a system. Moreover, signals over a network with limited information must be sampled and quantized. All of these bring some new challenges in the system analysis and controller design. The corresponding work has formed an active research area and various quantization schemes have been developed [10–22]. In [11], it is shown that a feedforward system can be stabilized by a coded feedback while satisfying some proper conditions. Ref. [13] presents a new tool for the quantized nonlinear control design, in which cyclic-small-gain theorem is employed to guarantee the stability property of the closed-loop quantized system. In [17], the feedback stabilization of nonlinear systems with quantized signals is considered under the assumption that a nominal feedback law is ISS with respect to a quantization error. This assumption is natural for a

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general nonlinear control system. Some results on designing such control laws are available and effective [5–9,23]. For example, the result of [23] shows that if a one-dimensional affine control system is stabilizable by means of a feedback, then it can be made ISS with respect to measurement disturbances by a proper feedback. However, it is difficult to test in general if a feedback nominal system is ISS with respect to measurement disturbances.

Motivated by the above with communication constraints, this paper focuses on the quantized feedback stabilization for a class of particular control systems in the form of Lur'e (i.e. the right hand sides are split into a linear part and a nonlinear part). Such a system has been well studied without information constraints. Our major concern is that if stabilization can be achieved for such a control system under information constraints, in which how to design a controller with a proper quantizer is a key factor. In this paper, we adopt a quantization algorithm, motivated by Hespanha et al. [10], which has some privilege of reducing the number of bits needed by quantizing an estimation error instead of the system state itself. Consequently, the required bandwidth is not so strictly restricted. However, different from [10], the quantizer adopted here is dynamic, rather than static, its parameters are time varying by employing a zoom variable. In this case, it shows that transmission error can be exponentially closed to zero, rather than only bounded like [10]. The effectiveness of the proposed quantization scheme is explicitly shown. Finally, the sufficient condition is given such that the control law for the asymptotical stabilization of the control system is obtained in terms of linear matrix inequalities (LMI) via a Lyapunov function.

This paper is arranged as follows. The problem formulation is given in Section 2 after an introduction in Section 1. The quantization algorithm is provided properly in Section 3. In Section 4, the main result is presented with a rigorous proof based on the designed quantization algorithm and the control law. Thereafter, a single-link flexible joint robot example with simulation is applied to show the obtained results. Finally, the summary is concluded.

Notation. Throughout this paper, we denote by $|\cdot|$ the standard Euclidean norm in R^n and by $\|\cdot\|$ the corresponding induced matrix norm in $R^{n\times n}$; the superscript "T" represents the transpose; "$*$" denotes the symmetric part; $X \geq Y$ (or $X > Y$ respectively) where X and Y are symmetric matrices, means that $X - Y$ is positively semi-definite (or positively definite respectively); Matrices, if not explicitly stated, are assumed to have compatible dimensions; A function $\gamma : R_{\geq 0} \to R_{\geq 0}$ is said to be of class $\mathcal{K}$ if it is continuous, strictly increasing, and $\gamma(0) = 0$; A function $\beta : R_{\geq 0} \times R_{\geq 0} \to R_{\geq 0}$ is said to be of class $\mathcal{KL}$ if, for each fixed $t \geq 0$, the function $\beta(\cdot, t)$ is a $\mathcal{K}$-function, and for each fixed $s \geq 0$, the function $\beta(s, \cdot)$ is decreasing and $\beta(s, t) \to 0$ as $t \to \infty$.

2. Problem formulation

We are interested in the following continuous-time system with nonlinear disturbance:

$$\dot{x} = Ax + Bu + f(t, x), \tag{1}$$

where $x \in R^n$ is the system state, $u \in R^r$ is a control input. A and B are the known constant matrices with appropriate dimensions. $f \in R^n$ is a vector-valued time-varying nonlinear perturbation with $f(t, 0) = 0$ for all $t \geq 0$ and satisfies a Lipschitz type condition for all $(t, x), (t, \hat{x}) \in R \times R^n$:

$$|f(t, x) - f(t, \hat{x})| \leq \alpha |F(x - \hat{x})|, \tag{2}$$

where F is a constant matrix with appropriate dimensions, α is a positive scalar. Consequently, from (2), we have

$$|f(t, x)| \leq \alpha |Fx|. \tag{3}$$

Remark 1. The structure of the perturbation in the form of (3) has been extensively discussed for both continuous and discrete time systems in the literature [3,4] and the references therein. It is worth mentioning that the matched uncertainty can be regarded as a special case of (3). However, the robust stability with a quantized feedback for (1) has seldom been investigated.

In this paper, we restrict ourselves to control functions in the form of static linear feedback

$$u = Kx, \tag{4}$$

where K is a controller gain matrix.

Suppose that the state of system (1) is sampled at time instants $t_k = kT_s\ k \in N = \{0, 1, 2, \dots\}$, where $T_s > 0$ is a sampling period. To reduce the required bits and bandwidth, we place the copy of the system (1) inside the coder and decoder so as to construct an estimate $\hat{x}$ of the system's state x.

$$\dot{\hat{x}} = A\hat{x} + Bu + f(t, \hat{x}), \quad u = K\hat{x},\ \hat{x}(0) = 0. \tag{5}$$

We denote by $e = x - \hat{x}$ as a transmission error and by $e_q = q(e)$ as a quantization error, where $q(\cdot)$ is a quantizer. In both coder and decoder, the state estimate $\hat{x}$ is updated by

$$\hat{x}(t_k) = \hat{x}(t_k^-) + q(e(t_k^-)) = \hat{x}(t_k^-) + q(x(t_k) - \hat{x}(t_k^-)), \tag{6}$$