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Controllability of non-densely defined impulsive neutral functional differential systems with infinite delay in Banach spaces

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ABSTRACT

In this paper, we investigate the controllability for a class of abstract impulsive neutral functional differential systems with infinite delay where the linear part is non-densely defined and satisfies the Hille–Yosida condition. The approach used is the Schauder fixed point theorem combined with the operator semigroups. Particularly, the compactness of the operator semigroups is not needed in this article.

1. Introduction

Various evolution processes, optimal control models in economics, stimulated neural networks, frequency-modulated systems and some motions of missiles or aircrafts are characterized by the fact that at certain moments of time they experience changes of state abruptly. The study of such dynamical systems with impulsive effects is of great importance. Nowadays, there has been increasing interest in the analysis and synthesis of impulsive systems, or impulsive control systems, due to their significance in both theory and applications; see [1–7] and the references therein.

It is well known that the issue of controllability plays an important role in control theory and engineering [5,8–10] because they have close connections to pole assignment, structural decomposition, quadratic optimal control, observer design etc. In recent years, the problem of controllability for various kinds of differential and impulsive differential systems has been extensively studied by many authors [11–17,9,18] using different approaches. Recently, Chang et al. [4] studied the controllability of impulsive neutral functional differential systems with infinite delay in Banach spaces by using Dhage's fixed point theorem. More recently, Chang et al. [3] proved the existence of solutions for non-densely defined neutral impulsive differential inclusions with nonlocal conditions by using the Leray–Schauder theorem of the alternative for kakutani maps.

In all these works [11,12,14,9,18], the linear operator A is always defined densely in X and satisfies the Hille–Yosida condition so that it generates a C_0-semigroup or analytic semigroup. However, as indicated in [19], we sometimes need to deal with non-densely defined operators. For example, when we look at a one-dimensional heat equation with Dirichlet conditions on $[0, 1]$ and consider $A = \frac{\partial^2}{\partial x^2}$ in $C([0, 1], R)$, in order to measure the solutions in the sup-norm, then the domain

$$D(A) = \{x \in C^2([0, 1], R); x(0) = x(1) = 0\},$$

is not dense in $C([0, 1], R)$ with the sup-norm. See [19] for more examples and remarks concerning non-densely defined operators. Motivated by [3,4], the main purpose of this paper is to study the controllability of impulsive functional differential

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systems with infinite delay defined non-densely. More precisely, we consider the controllability problem of the following impulsive neutral system with infinite delay on a general Banach space X:

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t) - g(t, x_t)] = A[x(t) - g(t, x_t)] + F(t, x_t) + Cu(t), \tag{1.1}$$

$$t \in J = [0, b], \quad t \neq t_k, \quad k = 1, 2, \ldots, m,$$

$$\Delta x|_{t=t_k} = I_k(x(t_k^-)), \quad k = 1, 2, \ldots, m, \tag{1.2}$$

$$x(t) = \varphi(t) \in \mathcal{B}_h, \tag{1.3}$$

where the state variable $x(\cdot)$ takes values in Banach space X with the norm $|\cdot|$ and the control function $u(\cdot)$ is given in $L^2(J, U)$, the Banach space of admissible control functions with U a Banach space. C is a bounded linear operator from U into X, the unbounded linear operator A is not defined densely on X, that is $\overline{D(A)} \neq X$.$F : J \times \mathcal{B}_h \to X$, $g : J \times \mathcal{B}_h \to X$, $I_k : X \to \overline{D(A)}$, $\Delta x|_{t=t_k} = x(t_k^+) - x(t_k^-)$ $(k = 1, 2, \ldots, m)$, $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = b$. Here $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t = t_k$, respectively. The histories $x_t : (-\infty, 0] \to X$, defined by $x_t(\theta) = x(t + \theta)$, $\theta \leq 0$, belongs to some abstract phase space $\mathcal{B}_h$.

Since the system (1.1)–(1.3) has impulsive effects, the phase space used in [20,21] cannot be applied to the above system. Here by using the technique in [22], we introduce an abstract phase space $\mathcal{B}_h$ in preliminaries. By using Schauder fixed point theorem, we establish a controllability result of the system (1.1)–(1.3) without the boundedness condition on impulsive functions I_k $(k = 1, 2, \ldots, m)$.

This paper has four sections. In the next section we introduce the technical framework under which our results are established. In the Section 3 we establish the controllability results of integral solutions for the system (1.1)–(1.3), can trivially be adjusted to include the infinite-dimensional space setting, if we replace the compactness of operators with the complete continuity of the nonlinearity. And also, two interesting corollaries can be obtained from our theorem. Finally, Section 4 is reserved for example.

2. Preliminaries

In this section, we shall introduce some basic definitions, notations and lemmas which are used throughout this paper.

At first, we present the abstract phase space $\mathcal{B}_h$. Assume that $h : (-\infty, 0] \to (0, +\infty)$ is a continuous function with $l = \int_{-\infty}^{0} h(t)\mathrm{d}t < +\infty$. For any $a > 0$, we define

$$\mathcal{B} = \{\psi : [-a, 0] \to X \text{ such that } \psi(t) \text{ is bounded and measurable}\},$$

and equip the space $\mathcal{B}$ with the norm

$$\|\psi\|_{[-a,0]} = \sup_{s \in [-a,0]} |\psi(s)|, \quad \forall\, \psi \in \mathcal{B}.$$

Let us define

$$\mathcal{B}_h = \left\{\psi : (-\infty, 0] \to X \text{ such that for any } c > 0,\ \psi|_{[-c,0]} \in \mathcal{B} \text{ and } \int_{-\infty}^{0} h(s)\|\psi\|_{[s,0]}\mathrm{d}s < +\infty\right\}.$$

If $\mathcal{B}_h$ is endowed with the norm

$$\|\psi\|_{\mathcal{B}_h} = \int_{-\infty}^{0} h(s)\|\psi\|_{[s,0]}\mathrm{d}s, \quad \forall\, \psi \in \mathcal{B}_h,$$

then it is clear that $(\mathcal{B}_h, \|\cdot\|_{\mathcal{B}_h})$ is a Banach space.

Now we consider the space

$$\mathcal{B}_h' = \{x : (-\infty, b] \to X \text{ such that } x_k \in C(J_k, X) \text{ and there exist } x(t_k^+) \\ \text{and } x(t_k^-) \text{ with } x(t_k) = x(t_k^-), x_0 = \varphi \in \mathcal{B}_h, k = 0, 1, \ldots, m\},$$

where x_k is the restriction of x to $J_k = (t_k, t_{k+1}]$, $k = 0, 1, \ldots, m$. Set $\|\cdot\|_h'$ be a seminorm in $\mathcal{B}_h'$ defined by

$$\|x\|_h' = \|\varphi\|_{\mathcal{B}_h} + \sup\{|x(s)| : s \in [0, b]\}, \quad x \in \mathcal{B}_h'.$$

To set the framework for our main controllability result, we will make use of the following definitions and lemma.

Definition 2.1 ([23]). Let X be a Banach space. An integrated semigroup is a family of operators $\{T_0(t)\}_{t \geq 0}$ of bounded linear operators $T_0(t)$ on X with the following properties:

(i) $T_0(0) = 0$;
(ii) $t \to T_0(t)$ is strongly continuous;
(iii) $T_0(s)T_0(t) = \int_0^s (T_0(t + r) - T_0(r))\mathrm{d}r$, for all $t, s \geq 0$.