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Chaotic behavior analysis based on sliding bifurcations

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ABSTRACT

In this paper, a mathematical analysis of a possible way to chaos for bounded piecewise smooth systems of dimension 3 submitted to one of its specific bifurcations, namely the sliding ones, is proposed. This study is based on period doubling method applied to the relied Poincaré maps.

1. Introduction

Piecewise smooth systems (p.w.s) considered in this paper are those defined by at least two sets of ordinary differential equations and switch from a smooth phase space to another one when some dynamic switching conditions occur, the sets that separate those phase spaces are smooth and of codimension one. Many special phenomena are relied to those systems (in addition to the classic ones) particularly the sliding ones; recall that those phenomena are well known and studied since at least the famous Filippov book [1]. Moreover, the Russian school has also proposed pioneer works in piecewise smooth bifurcations [2,3]. More recently, Mario di Bernardo and coauthors gave a bifurcation's classifications for piecewise smooth systems [4–6] using the relied Poincaré maps. On the basis of this special classifications, we analyze the possibility of generating chaos for bounded p.w.s systems via these kinds of bifurcations; we have already presented some results in [7,8] concerning respectively the grazing bifurcation's and the corner bifurcation's cases. Hereafter, we investigate the possibility of generating chaos for bounded p.w.s systems of dimension 3 submitted to sliding bifurcations. Obviously, these kinds of piecewise smooth systems and corresponding bifurcation's analysis already exists [9,10], nevertheless, at our knowledge, a general mathematical study of a way to chaos via this type of bifurcations has never been proposed. Consequently, in this paper, following the analysis given in [11,12] and based on Lyapunov–Schmidt method, a solution analysis is proposed for generating chaotic behavior, this will be done using period doubling method.

The paper is organized as follows: In Section 2, the problem statement is established; in Section 3, the problem analysis is developed, a way to chaos is proposed in Section 4 and an academic example based on this approach with simulation results are presented in Section 5.

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2. Recalls and problem statement

Let us consider the following piecewise smooth system:

$$\dot{x} = \begin{cases} F_1(x) & \text{if } H(x) \geq 0 \\ F_2(x) & \text{if } H(x) < 0 \end{cases} \tag{1}$$

where $x : I \longrightarrow D$, D is an open bounded domain of R^3, $I \subset R^+$, generally I is the time interval.

$$F_1, F_2 : C^1(I, D) \longrightarrow C^k(I, D), \quad \text{for } k \geq 4$$

where $C^k(I, D)$ is the set of C^k functions defined on I and having values in R^3, the norm for $C^k(I, D)$ is defined as follows:

$$x \in C^k(I, D) : \|x\| = \sup_{t\in I} \|x(t)\|_e + \sup_{t\in I} \|\dot{x}(t)\|_e + \cdots + \sup_{t\in I} \left\|x^{(k)}(t)\right\|_e$$

where $x^{(k)}(.)$ denotes the kth derivative of $x(.)$ and $\|.\|_e$ is a norm defined on R^3.

According to [13], $(C^k(I, D), \|.\|)$ is a Banach space.

Thus, both vector fields F_1 and F_2 are sufficiently regular on their domains of definitions such that the flows Φ_i, $i = 1, 2$ generated by each vector field F_i are defined as the operators that satisfy:

$$\frac{\partial \Phi_i(x, t)}{\partial t} = F_i(\Phi_i(x, t)) \quad \text{and} \quad \Phi_i(x, 0) = x, \quad i = 1, 2.$$

$H : D \longrightarrow R$ is a C^1 application, it is a phase space boundary between both regions of smooth dynamics, H defines the set:

$$S = \{x(t) \in D : H(x(t)) = 0\}$$

and S is the switching manifold and divides the phase space into two regions:

$$S^+ = \{x(t) \in D : H(x(t)) > 0\}$$
$$S^- = \{x(t) \in D : H(x(t)) < 0\}.$$

Moreover for being in the context of sliding phenomena, it is assumed that there exists a subset of switching manifold $\bar{S} \subset S$ denoted as sliding region which is simultaneously attracting from $S^+$and S^-, so, considering any neighborhood $\upsilon_{\bar{S}}$ of $\bar{S}$, the existence of $\bar{S}$ is characterized by the following hypothesis:

$$(\mathrm{H}_1) \quad \langle \nabla H(x(t)), F_2(x(t)) - F_1(x(t))\rangle \in R_+^*, \quad \text{for all } x(t) \in \upsilon_{\bar{S}}.$$

where $\langle ., .\rangle$ is a usual scalar product on R^3.

Thus, under (H_1), if the system trajectory crosses $\bar{S}$, the sliding motion evolves in $\bar{S}$ until it eventually reaches its boundary, this motion can also be described by considering an appropriate vector $F_{\bar{S}}$ witch lies within the convex hull of F_1 and F_2 and is tangent to $\bar{S}$ for each $x(t) \in \upsilon_{\bar{S}}$, $F_{\bar{S}}$ is given by:

$$F_{\bar{S}}(x(t)) = \frac{F_1(x(t)) + F_2(x(t))}{2} + H_u(x(t))\frac{F_1(x(t)) - F_2(x(t))}{2},$$

where $H_u : D \longrightarrow R$ is deduced from the equivalent vector method [1,14], in fact H_u is given using the tangential condition of $F_{\bar{S}}$ on the switching manifold i.e. $\langle \nabla H, F_{\bar{S}}\rangle = 0$ which implies that :

$$H_u(x(t)) := -\frac{\langle \nabla H(x(t)), F_1(x(t)) + F_2(x(t))\rangle}{\langle \nabla H(x(t)), F_1(x(t)) - F_2(x(t))\rangle} \tag{2}$$

and

$$-1 \leq H_u(x(t)) \leq 1, \quad \text{for } x(t) \in \upsilon_{\bar{S}}. \tag{3}$$

So, the sliding region is redefined as :

$$S_\upsilon = \left\{x(t) \in \upsilon_{\bar{S}} : -1 \leq H_u(x) \leq 1\right\}. \tag{4}$$

It follows from (H_1) and (4) that sliding phenomena is characterized by:

$$\langle \nabla H(x(t)), F_2(x(t))\rangle > 0 > \langle \nabla H(x(t)), F_1(x(t))\rangle \quad \text{for } x(t) \in \upsilon_{\bar{S}} \tag{5}$$

and the boundary of the sliding region is defined as:

$$\partial S_\upsilon^+ = \{x(t)\epsilon S_\upsilon : H_u(x(t)) = 1\} \quad \text{and} \quad \partial S_\upsilon^- = \{x(t)\epsilon S_\upsilon : H_u(x(t)) = -1\}.$$

The system (1) characterizes the behavior of an increasing number of dynamic systems particularly in Applied Sciences and Engineering, those piecewise smooth systems can present a specific switching bifurcations particularly when depending